THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE fourteenth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of California, Saturday, September 26. The following eleven members of the Society were present: Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor J. H. McDonald, Professor H. C. Moreno, Professor C. A. Noble, Professor T. M. Putnam, Mr. H. W. Stager, Mr. J. D. Suter.

Professor D. N. Lehmer was elected to succeed Professor Hoskins as chairman of the Section. The present Secretary was reëlected, and Professors Blichfeldt and McDonald were named to serve with the secretary on the program committee, Dr. Blichfeldt being designated as chairman of the committee.

The two meetings in 1909 will be held on February 27, at Stanford University, and on September 25, at the University of California.

The following papers were read at this meeting:

(1) Professor H. F. BLICHFELDT: "A theorem on simple groups."

(2) Professor J. H. MCDONALD: "On Fourier series and expansions in spherical harmonics."

(3) Professor H. C. MORENO: "On a class of ruled loci."

(4) Dr. ARTHUR RANUM: "The general term of a recurring series."

(5) Professor C. A. NOBLE: "Necessary conditions that three or more partial differential equations of the second order shall have common solutions."

Abstracts of these papers are given below in the order of their presentation.

1. Professor Blichfeldt's paper is in substance as follows: Let F be a group of order $p^\lambda n$, p being prime to n, containing a Sylow subgroup G of order p^λ. This has a subgroup G_1 of order $p^{\lambda-1}$. Let S be one of the substitutions of lowest order contained in G but not in G_1. Then, either F will contain an invariant subgroup of index p, or there is in G a subgroup K

containing at least every substitution A of G for which $SAS^{-1}A^{-1}$ is an invariant substitution of G_1, — this group K being transformed into itself by a substitution T of H, though T does not transform S into itself. The order of T is prime to p. Further properties of K were stated; e. g., $K = G$ when the latter contains no substitutions of order p^2. The types of certain Sylow subgroups G contained in a simple group H were shown to be restricted by this theorem. Cf. Frobenius, "Über auflösbare Gruppen, V," *Berliner Sitzungsberichte*, 1901, pages 1324–1329.

2. Professor McDonald's paper discusses the application of Poisson's integral to the summation of a Fourier series and investigates Laplace's method of expansion in spherical harmonics in a similar way.

3. The general problem of finding, in a space of n dimensions, the locus of all the lines meeting n $(n-2)$-flats was presented by Professor Moreno at a previous meeting of this section. In the present paper, the same problem for $n = 4$ is treated more in detail. By a special choice of coordinates the equation of the 3-spread was shown to be $\Sigma xyz + n^2\Sigma x = 0$. This is Segre's cubic spread. In this form of the equation, many properties of the spread are readily deduced.

4. Let $u_0 + u_1 + u_2 + \cdots$ be a recurring series of the nth order, in which every term after the nth is expressed as a linear function of the n preceding terms by means of the formula $u_m = a_1 u_{m-1} + \cdots + a_n u_{m-n}$. By successive applications of this formula the general term is expressible as a linear function of the first n terms, and a rational integral function of the n constants of the scale, say $u_m = F_m(u_0, \cdots, u_{n-1}; a_1, \cdots, a_n)$. Nevertheless, the classical method of finding it involves the irrational process of decomposing the generating function of the series into partial fractions. In this paper Dr. Ranum gives a rational method and determines the explicit form of the function F_m. As an application, the mth power of a linear substitution in n variables is expressed in terms of its first $n-1$ powers.

5. Following a suggestion made by Hilbert in his lectures in 1900, and which was carried out in some detail by Yoshiye

in volume 57 of the *Annalen*, Professor Noble extends the results of Yoshiye by finding the necessary conditions that three or more equations of the form

$$F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}\right) = 0$$

shall have solutions in common.

W. A. MANNING,
Secretary of the Section.

NOTE ON STATISTICAL MECHANICS.

BY PROFESSOR EDWIN BIDWELL WILSON.

(Read before the American Mathematical Society, September 11, 1908.)

IN developing the elements of statistical mechanics it is customary though by no means essential to remark the analogy between that subject and hydromechanics.* The analogy arises primarily through the fact that the equation of continuity of hydrodynamics,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{or} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

where u, v or u, v, w are the velocities in the fluid in two or in three dimensions, exists in the form

$$(1) \qquad \frac{\partial \dot{p}_i}{\partial p_i} + \frac{\partial \dot{q}_i}{\partial q_i} = 0 \quad \text{or} \quad \sum_{i=1}^{i=n} \frac{\partial \dot{p}_i}{\partial p_i} + \frac{\partial \dot{q}_i}{\partial q_i} = 0$$

for dynamical systems regulated by the hamiltonian canonical equations

$$(2) \qquad \dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \quad (i = 1, 2, \cdots, n);$$

and it is further exemplified by connection of the Boltzmann-Larmor hydrodynamical interpretation of Jacobi's last multiplier with the principle of conservation of extension in phase.† The object of this note is to comment upon the analogy in question.

* Jeans, The Dynamical Theory of Gases, p. 62. Gibbs, Elementary Principles in Statistical Mechanics, p. 11.

† Compare Whittaker, Analytical Dynamics, p. 272, and Gibbs, loc. cit., p. 29.

Let the $2n$ quantities $q_1, q_2, \cdots, q_n, p_1, p_2, \cdots, p_n$ be interpreted as rectangular coordinates in a space of $2n$ dimensions. And suppose that an ensemble of systems which satisfy (2) be imagined as distributed in that space with a density in phase D or with a probability in phase P which are functions of p_i, q_i and the time t. Then, by the same reasoning as in hydromechanics, the density D or the probability P will satisfy the equation

$$\frac{dP}{dt} + \dot{P}\sum_{i=1}^{i=n}\left(\frac{\partial \dot{p}_i}{\partial p_i} + \frac{\partial \dot{q}_i}{\partial q_i}\right) = \frac{dP}{dt} = 0.$$

As the condition for statistical equilibrium is that the density shall be independent of the time, it follows that for statistical equilibrium

$$\frac{\partial P}{\partial t} = -\sum_{i=1}^{i=n}\left(\frac{\partial P}{\partial p_i}\dot{p}_i + \frac{\partial P}{\partial q_i}\dot{q}_i\right) = 0.$$

These results are obtained with full discussion by Gibbs, loc. cit.

Suppose that the systems considered are specified by a single coordinate q and the momentum p. The connected hydrodynamical problem thus becomes two-dimensional. Let it be asked whether the fluid which represents the ensemble of systems is more fully comparable with the ordinary fluid of hydrodynamics than is implied by the existence of an analogue for the equation of continuity. In the first place it should be noted that the motion of the fictitious fluid is given by equations relating to its velocities, that is, by

$$q = \frac{\partial H}{\partial p} = u(p, q), \quad \dot{p} = \frac{\partial H}{\partial q} = v(p, q).*$$

If the velocities of a particle in ordinary dynamics be given, a differentiation with respect to the time will suffice to determine the forces. If the velocities in a continuous medium are given, the problem is not so simple. For suppose that

$$u = u(x, y) \quad \text{and} \quad v = v(x, y)$$

are the velocities in an ordinary fluid. Then

* For convenience and definiteness it will be assumed throughout this note that H does not depend explicitly upon the time and moreover that H may be decomposed into the sum of the kinetic and potential energies.

$$\frac{du}{dt} = \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial t} = X - \frac{1}{\rho}\frac{\partial p}{\partial x},$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial x}u + \frac{\partial V}{\partial y}v + \frac{\partial V}{\partial t} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y},$$

where ρ is the density, p is the pressure, and X and Y are the components of the impressed forces per unit mass. It therefore appears that ρ and p may be assigned at pleasure and X, Y determined to suit.

To remove this indeterminateness, it might be required that the forces X, Y be derivable from a potential. Then $\dot{u}$ and $\dot{v}$, known functions of the variables, would afford the two equations

$$\dot{u} = -\frac{\partial V}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x}, \quad \dot{v} = -\frac{\partial V}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial y}.$$

If ρ be constant, as in the dynamics of liquids, the problem is now impossible unless

$$\dot{u}dx + \dot{v}dy$$

is a perfect differential. On the other hand if ρ be not constant but a known function of the variables, the expression of the fact that

$$\left(\dot{u} + \frac{1}{\rho}\frac{\partial p}{\partial x}\right)dx + \left(\dot{v} + \frac{1}{\rho}\frac{\partial p}{\partial y}\right)dy$$

be a perfect differential will afford a partial differential equation from which p may be determined. It should be remembered, however, that in the case of any real fluid, there is generally assumed an equation $F(p, \rho, T) = 0$ connecting the pressure, density, and temperature. The temperature is generally disregarded for purposes of hydrodynamics and it is assumed that $F(p, \rho) = 0$. The determination of p by the method suggested above would not generally give a result in which p was a function of ρ. In fact if p is a function of ρ, the equations may be written as

$$\dot{u} = -\frac{\partial V}{\partial x} - \frac{\partial \bar{p}}{\partial x}, \quad \dot{v} = -\frac{\partial V}{\partial y} - \frac{\partial \bar{p}}{\partial y}, \quad \bar{p} = \int \frac{dp}{\rho}$$

and the problem would again generally be impossible.

With these points in connection with the purely hydrodynamical problem in mind we may return to the matter of statistical mechanics and ask in the first place what are the conditions that there should be a velocity potential for the motion of the fictitious fluid. If, when there is only one variable,

$$\dot{q}dq + \dot{p}dp = \frac{\partial H}{\partial p}dq - \frac{\partial H}{\partial q}dp$$

is a perfect differential, the expression

$$\Delta H = \frac{\partial^2 H}{\partial p^2} + \frac{\partial^2 H}{\partial q^2} = 0.$$

Now by hypothesis, H is of the second order in p. Hence

$$\frac{\partial^3 H}{\partial p^3} + \frac{\partial^3 H}{\partial p \partial q^2} = 0 \quad \text{gives} \quad \frac{\partial^3 H}{\partial p^3} = \frac{\partial^3 H}{\partial p \partial q^2} = 0.$$

The differentiation with respect to p and q to find the derivatives of the fourth order shows at once that

$$\frac{\partial^4 H}{\partial p^4} = \frac{\partial^4 H}{\partial p^3 \partial q} = \frac{\partial^4 H}{\partial p^2 \partial q^2} = \frac{\partial^4 H}{\partial q^4} = 0.$$

Hence H is a polynomial of at most the third degree in p and q.

If there had been more variables, say two, the conditions that

$$\frac{\partial H}{\partial p_1}dq_1 - \frac{\partial H}{\partial q_1}dp_1 + \frac{\partial H}{\partial p_2}dq_2 - \frac{\partial H}{\partial p_2}dp_2$$

be a perfect differential would be

$$\frac{\partial^2 H}{\partial p_1^2} + \frac{\partial^2 H}{\partial q_1^2} = 0, \qquad \frac{\partial^2 H}{\partial p_2^2} + \frac{\partial^2 H}{\partial q_2^2} = 0,$$

$$\frac{\partial^2 H}{\partial p_1 \partial p_2} + \frac{\partial^2 H}{\partial q_1 \partial q_2} = 0, \quad \frac{\partial^2 H}{\partial p_1 \partial q_2} = \frac{\partial^2 H}{\partial p_2 \partial q_1} = 0,$$

and in a similar manner would show that H must be a polynomial of at most the third degree in p_i and q_i jointly. The result is the same for any number of variables. Hence it appears that only in a very restricted type of cases can there exist a velocity potential for the fictitious fluid, namely, when H, being of the second degree in p_i, is at most of the third

degree in p_i and q_i jointly, and even in this case there are still certain relations to be satisfied by the coefficients of the polynomial H. In such cases only would a velocity potential exist for the fictitious motion.

Let it now be asked whether the equations

$$\ddot{q} = -\frac{\partial V}{\partial q} - \frac{1}{P}\frac{\partial \Pi}{\partial q}, \quad \ddot{p} = -\frac{\partial V}{\partial p} - \frac{1}{P}\frac{\partial \Pi}{\partial p},$$

where P is the probability in phase and Π is a fictitious pressure for the fictitious fluid, can be satisfied. In the first place it may be noted that

$$\ddot{q} = \frac{d}{dt}\frac{\partial H}{\partial p} = \frac{\partial^2 H}{\partial q \partial p}\frac{\partial H}{\partial p} - \frac{\partial^2 H}{\partial p^2}\frac{\partial H}{\partial q} = -J\left(H, \frac{\partial H}{\partial p}\right),$$

$$\ddot{p} = -\frac{d}{dt}\frac{\partial H}{\partial q} = -\frac{\partial^2 H}{\partial q^2}\frac{\partial H}{\partial p} + \frac{\partial^2 H}{\partial q \partial p}\frac{\partial H}{\partial q} = J\left(H, \frac{\partial H}{\partial q}\right),$$

where J denotes the jacobian with respect to q and p of the quantities in the parenthesis. The equations to be satisfied are therefore

$$-J\left(H, \frac{\partial H}{\partial p}\right) + \frac{1}{P}\frac{\partial \Pi}{\partial q} = -\frac{\partial V}{\partial q},$$

$$J\left(H, \frac{\partial H}{\partial q}\right) + \frac{1}{P}\frac{\partial \Pi}{\partial p} = -\frac{\partial V}{\partial p}.$$

The equation obtained by differentiating these expressions respectively with respect to p and q and subtracting is seen at once to be simply

$$J(H, \Delta H) = J(\Pi, P^{-1}). \tag{3}$$

This is a partial differential equation of the first order and first degree in Π which will in general suffice to determine Π when P is known.

The form of (3) is such that one interesting theorem is obvious: The necessary and sufficient condition that there may be a functional relation $F(\Pi, P) = 0$ between the pressure and density in the fictitious fluid is that the function H satisfy the relation

$$\Delta H = \frac{\partial^2 H}{\partial p^2} + \frac{\partial^2 H}{\partial q^2} = \Phi(H),$$

where Φ is any function of H. In this case $\ddot{q}dq + \ddot{p}dp$ is a perfect differential and the complete integration of the differential may be found.

$$\int\left(\frac{\partial^2 H}{\partial q \partial p}\frac{\partial H}{\partial p} - \frac{\partial^2 H}{\partial p^2}\cdot\frac{\partial H}{\partial q}\right)dq + \left(\frac{\partial^2 H}{\partial q \partial p}\frac{\partial H}{\partial q} - \frac{\partial^2 H}{\partial q^2}\frac{\partial H}{\partial p}\right)dp =$$

$$\frac{1}{2}\left[\left(\frac{\partial H}{\partial p}\right)^2 + \left(\frac{\partial H}{\partial q}\right)^2\right] - \int \Phi(H)\,dH = -V - \overline{\Pi},$$

where

$$\overline{\Pi} = \int \frac{d\Pi}{P} = \int \frac{d\Pi}{dH}\frac{dH}{P}.$$

It therefore appears natural to take the functions V and Π as

$$V = -\tfrac{1}{2}\left[\left(\frac{\partial H}{\partial p}\right)^2 + \left(\frac{\partial H}{\partial q}\right)^2\right], \quad \Pi = \int P\,\Phi(H)dH.*$$

Here P may be any function of H which satisfies the necessary relation

$$\int\int P dp dq = 1 \quad \text{(within limits considered)},$$

although there is a large family of functions defined by such a general relation as $\Delta H = \Phi(H)$. The fact that H is necessarily quadratic in p suffices, as in the case of $\Delta H = 0$, to determine H except for a few arbitrary constants of integration. In fact, under the assumption that H is divisible into a kinetic and a potential energy,

$$H = \tfrac{1}{2}f(q)p^2 - \phi(q), \quad \Delta H = f(q) + \tfrac{1}{2}f''(q)p^2 - \phi''(q),$$

$$J(H, \Delta H) = \begin{vmatrix} \tfrac{1}{2}f'(q)p^2 - \phi'(q) & f(q)p \\ f'(q) + \tfrac{1}{2}f'''(q)p^2 - \phi'''(q) & f''(q)p \end{vmatrix} \equiv 0,$$

and hence

$$f(q)f'''(q) - f''(q)f'(q) = 0,$$

$$f(q)[f'(q) - \phi'''(q)] + f''(q)\phi'(q) = 0.$$

The first of these equations gives $f''' = kf$ and the second reduces to

$$f'(q) - \phi'''(q) + k\phi'(q) = 0.$$

*This choice seems the simplest; but any function of H may be added to Π and a compensating change introduced in V.

The complete determination of f then gives the results

$$f = aq + b, \quad f = a\cos(\sqrt{-k}q + b), \quad f = ae^{\sqrt{k}q} + be^{-\sqrt{k}q},$$

according as k is zero, negative or positive. The determination of $\phi(q)$ then calls for the solution of a linear equation with constant coefficients and with second member. The class of problems of mechanics which result from such a value of H is somewhat restricted. In case the coordinate q is so chosen that $f(q) = \text{const}$, $\phi'''(q) = 0$; and the potential energy is a quadratic in q — a very restricted but well known and frequently occurring type of problem.*

To return to the general case, $J(H, \Delta H) \neq 0$. Here the equation (3) will determine Π when P is given. As there is only one variable, there is only one invariant of the motion of any given one of the systems in the ensemble, and that invariant is H. For statistical equilibrium, it is therefore necessary that P shall be a function of H. As H and ΔH are functionally independent, they may be taken as the independent variables, and by the rules of operation with jacobians,

$$1 = \frac{J(\Pi, P^{-1})}{J(H, \Delta H)} = \begin{vmatrix} \frac{\partial \Pi}{\partial \Delta} & \frac{\partial P^{-1}}{\partial H} \\ \frac{\partial \Pi}{\partial \Delta H} & \frac{\partial P^{-1}}{\partial \Delta H} \end{vmatrix}$$

Hence for statistical equilibrium this result becomes merely

$$\frac{dP^{-1}}{dH}\frac{\partial \Pi}{\partial \Delta H} = -1, \quad \Pi = -\frac{dH}{dP^{-1}}\Delta H + f(H).\dagger$$

The expression for Π is determined except for an additive arbitrary function of H. No more complete determination could be expected because the fictitious potential V may take on an additive function of H.

For simplicity it may be assumed that the additive function $f(H)$ is omitted from Π. The determination of V is then straightforward and simple. In fact

* It may be noted that if $\Delta H = 0$ in this case, the potential energy is $-\phi(q) = -\frac{1}{2}f(q)q^2 + mq + n$, where $f(q)$ is a constant necessarily positive. The force is therefore repulsive, and Gibbs's canonical distribution would be illusory. See Gibbs, loc. cit., p. 35.

† This expression which is of the form $\Pi = \phi(P)\Delta H + \psi(P)$ corresponds to the equation $F(p, \rho, T) = 0$ of ordinary fluids. In a way, therefore, it may be said that ΔH corresponds to T.

$$V = P^{-1}\Delta H \frac{dH}{dP^{-1}} - \frac{1}{2}\left[\left(\frac{\partial H}{\partial p}\right)^2 + \left(\frac{\partial H}{\partial q}\right)^2\right].$$

With this value of V and the value for Π obtained above the equations of the fictitious fluid become identical with those of a real fluid. The values of V and Π may further be written as

$$V = -\Delta H \frac{dH}{d\eta} - \frac{1}{2}\left[\left(\frac{\partial H}{\partial p}\right)^2 + \left(\frac{\partial H}{\partial q}\right)^2\right],$$

$$\Pi = e^{\eta}\Delta H \frac{dH}{d\eta}, \quad \text{where } \eta = \log P$$

is the index of probability in phase. As e^{η} is necessarily positive, $\Delta H \cdot dH/d\eta$ must be always positive if Π is to be positive, as in the case of a real fluid. In the case of Gibbs's canonical distribution

$$P = e^{\frac{\Psi - H}{\Theta}}, \quad \eta = \frac{\Psi - H}{\Theta}$$

and $d\eta/dH$ is negative. For simple vibrating systems ΔH is positive * and hence for these systems Π would be negative and represent a tension.

The difficulty that Π is frequently negative may be remedied in some cases by a proper choice of the additive function $f(H)$. This, however, complicates the formulas and is not particularly desirable. In fact if the fictitious medium tended to expand, the concept of statistical equilibrium would be difficult of application. On the whole, the hydrodynamic analogy does not appear to be as close and vital as could be desired when developed to a greater extent than is implied in the equation (1) of continuity, although tolerably satisfactory results have been obtained for the case $n = 1$. When the number of variables n is greater than one, the $2n$ equations

$$\ddot{q}_i = -\frac{\partial V}{\partial q_i} - \frac{1}{P}\frac{\partial \Pi}{\partial q_i}, \quad \ddot{p}_i = -\frac{\partial V}{\partial p_i} - \frac{1}{P}\frac{\partial \Pi}{\partial p_i}$$

cannot be satisfied except under extremely restricted hypotheses concerning H. The conditions that the expression

* For instance when $H = \frac{1}{2}p^2 + \frac{1}{2}n^2q^2 + kq^4$. If the system is that of harmonic vibrations, ΔH is constant, $J(H, \Delta H) = 0$, and the system comes under a previous type.

$$\Sigma\left(q_i+\frac{1}{P}\frac{\partial\Pi}{\partial q_i}\right)dq_i+\Sigma\left(p_i+\frac{1}{P}\frac{\partial\Pi}{\partial p_i}\right)dp_i=-dV$$

are too numerous to be satisfied by any functions P, Π, V. It is, however, possible that a medium, which satisfies the equation of continuity (1) and has equations more complicated than the hydromechanical type, some elastic medium, may be found to answer the purposes of the problem.

From a physical point of view, however, there is one matter which is of serious disadvantage in any hydrodynamical interpretation of the problem of statistical mechanics. That is the question of dimensions. If v denote the velocity in the fictitious fluid, then

$$v^2=\dot{q}_1^2+\dot{p}_1^2+\dot{q}_2^2+\dot{p}_2^2+\cdots$$

is without physical significance unless the dimensions of q_1, q_2, $\cdots$, p_1, p_2, $\cdots$ are all alike. But the very equations (2) defining q_ν p_ν give

$$[q_i]=\frac{[H]}{[p_i]},\quad [p_i]=\frac{[H]}{[q_i]},\quad [p_iq_i]=[H],\quad [p_i]=[q_i],$$

where the brackets are used as dimensional symbols. Hence the dimensions are

$$[p_i]=[q_i]=[m]^{\frac{1}{2}}\frac{[l]}{[t]}.$$

where m, l, t denote mass, distance, time. As p_i and q_i seldom, if ever, have such dimensions as are here required, v^2 is not a physical quantity. The same is true of ΔH. It is interesting to note that the equation of continuity is such that its dimensionality is always $[t]^{-1}$, as it should be.

The importance of the homogeneity of physical dimensions of formulas is so great and mathematical investigations and analysis carried on without due regard to dimensions are so seldom of physical importance, that despite the results obtained for $n=1$, the above considerations suggest the improbability of finding any continuous medium which shall properly shadow forth the relations of statistical mechanics. Otherwise it would be tempting to try to find such media for the cases where $n\geqq 2$. That which replaces the detailed physical concept of a medium is the mathematical concept of integral invariants, and these are dimensionally homogeneous.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS., *August*, 1908.

ON CERTAIN CONSTANTS ANALOGOUS TO FOURIER'S CONSTANTS.

BY DR. C. N. MOORE.

THROUGH the kindness of Professor Landau my attention has been called to the fact that Theorem I of the article of the above title which appeared in the BULLETIN * may be obtained from a theorem due to Lerch † by means of a change of variable and a slight modification of the form of statement.

With regard to Theorem II of the same article I wish to state that the interval $a \leqq x \leqq b$ must be less than 2π in order that the proof there given may hold. However, the theorem is still true when the interval in question is equal to 2π, and may be proved very simply from a theorem due to Fejér. The latter has shown ‡ that if in an interval of length 2π a function is finite save for a finite number of points, and is integrable, the Fourier development corresponding to that function is summable to the value of the function § at every point of that interval at which the function is continuous. Under the conditions of Theorem II the Fourier development corresponding to $\psi(x)$ in the interval $a \leqq x \leqq b$ has each of its terms equal to zero and is therefore summable to the value zero at every point of that interval. However, we know from Fejér's theorem that the development is summable to the value of the function $\psi(x)$ at every point of the interval $a \leqq x \leqq b$ at which this function is continuous and hence $\psi(x)$ is zero at every such point.

CAMBRIDGE, MASS.,
June, 1908.

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* BULLETIN, ser. 2, vol. 14 (1908), p. 368.

† *Acta Mathematica*, vol. 27 (1903), p. 345.

‡ *Math. Annalen*, vol. 58 (1903–04), p. 59.

§ *I. e.*, the limit as n becomes infinite of the arithmetic mean of $s_1, s_2, \cdots, s_n$, where s_m represents the sum of the first m terms of the series, exists and is equal to the value of the function.

THE COLOGNE MEETING OF THE DEUTSCHE MATHEMATIKER-VEREINIGUNG.

THE eightieth convention of German naturalists and physicians was held at Cologne, September 20–26, 1908. Mathematics and astronomy together formed one of the thirty-one sections. The total attendance was about two thousand, of which the Deutsche Mathematiker-Vereinigung as a sub-division claimed nearly one hundred.

On the evening of September 20, an informal reception was given in the Bürgergesellschaft. A general session of all the divisions took place in the Gürzenich on the following morning. After greetings from representatives of the administration and from various societies in Cologne, two papers were read: (1) The achievements and influence of Albertus Magnus (one of the founders of the ancient Cologne University); (2) Motor balloons and flying machines.

The sessions of the Vereinigung were held in the Handels-Hochschule. Under the influence of President Klein, the society devoted the meetings largely to the problems of applied mathematics, thus acting as mentor for the Naturforscherversammlung. The following papers were read:

1. H. MINKOWSKI, Göttingen, "Space and time."
2. G. HAMEL, Brünn, "The foundations of mechanics."
3. E. H. TIMERDING, Strassburg, "The historical development of the concept of force."
4. P. STÄCKEL, Karlsruhe, "Remarkable motions of the top."
5. R. v. MISES, Brünn, "Problems of technical hydromechanics."
6. F. MÜLLER, Dresden, "Plans for the publication of the works of Leonhard Euler."
7. H. WIENER, Darmstadt, "On the geometry of the binary forms."
8. H. JUNG, Marburg, "Algebraic functions of two independent variables."
9. H. REISSNER, Aachen, "Scientific problems in the mechanics of flight."

Number 8 has already appeared in the *Comptes Rendus;* numbers 2 and 4 will soon be published in the *Mathematische Annalen*, and number 5 in the *Physikalische Zeitschrift.* Ab-

stracts of the other papers are given below, the numbers corresponding to those in the list above.

1. Professor Minkowski discussed the complete revolution that has taken place in our conception of time and space, owing to the exact mathematical deductions from the latest investigations of physics. Lagrange has called physics a four-dimensional geometry because three dimensions of space, and one of time are introduced, and this definition seems to-day to be applicable in a deeper, unhoped-for sense. Minkowski shows that the remarkable hypothesis of H. A. Lorentz concerning the contraction of the electrons and the alleged contradiction between the newtonian mechanics and the modern theory of electricity can be completely accounted for if we assume that we live in a four-dimensional world of which the fourth dimension (time) may be neglected with a certain amount of freedom. The axiom that the velocity of matter cannot exceed that of light in ether plays here an important rôle. This new comprehension of the world as a sort of union of time and space makes possible great strides in the theory of electricity and magnetism, and requires finally a revision of all physical theories. That these revisions are capable of being carried out is due alone to the many-sided advances made by pure mathematics in the past century.

3. Professor Timerding's paper was an historical review of the mathematical conception of force with especial reference to the notions of Newton.

6. Professor Müller sketched the career and influence of Euler and outlined the plans that are being made for the publication of his works.

7. Professor Wiener continued his investigations in the geometric interpretation of binary forms, making use of polarity and reflection.

9. There are three chief properties of a flying machine, the capability of floating, the stability, and the power. The simplest cases of the first can be solved by theoretical hydrodynamics, but the more difficult must be experimentally determined. The problem of the most favorable shape of the planes and the most advantageous angle of motion has not been systematically treated since the time of Lilienthal, and our knowl-

edge regarding the position of the resulting pressure is very incomplete. It is shown that the problem of stability must be treated as a case of oscillation about a state of stationary motion. The vibration in the plane of motion may be regarded as consisting of two components, of which one however is so damped that it does not come into practical computation. In a machine consisting of parallel surfaces the presence of horizontal tail surfaces aids in lessening the oscillation, but they are not economical when the power comes into consideration. Models are not helpful in stability experiments because the theory of dimensions is not applicable. A strictly hydrodynamical treatment of the propeller problem is not at present possible, but the application of the laws of oblique motion of thin plates produces a good propeller. Professor Reissner has found also that the propeller which he constructed in accordance with his solution of the problem by means of the calculus of variations works to good advantage, and his researches in cooperation with Professor Junkus are to be continued on a larger scale.

The business session of the Deutsche Mathematiker-Vereinigung was held on the afternoon of the 22nd. The report showed that the present membership is about seven hundred. The executive committee for the year 1908–09 is as follows: M. Krause, president; A. Krazer, secretary; A. Ackermann-Teubner, treasurer; A. Gutzmer, editor of the *Jahresbericht;* A. Schönflies, F. Klein, F. Engel, and F. Schur. As German representatives in the international committee on mathematical instruction to report to the congress in Cambridge (1912), F. Klein and P. Treutlein were chosen. The society voted the sum of five thousand francs in aid of the publication of the works of Euler undertaken by the Swiss scientific societies.

Professors Bolza, of Chicago, Field, of Michigan, and Richardson, of Brown were the only Americans present. On the evening of the 21st the visitors were entertained in the zoological gardens by the scientific societies of Cologne, and on the following evening a banquet was held in the Gürzenich. On Wednesday evening the theatre and opera gave special performances to which the members were invited. The reception given by the city took place on Thursday evening, and on Saturday four excursions to various points of interest in the vicinity of Cologne were arranged. The next meeting will be held in Königsberg.

R. G. D. RICHARDSON.

GOURSAT'S COURS D'ANALYSE.

Cours d'Analyse mathématique. By ÉDOUARD GOURSAT, Professeur à la Faculté des Sciences de Paris. Vol. II, *Theory of Analytic Functions; Differential Equations, Total and Partial; Elements of the Calculus of Variations.* Paris, Gauthier-Villars, 1905. 8vo, pp. vi + 640.

THE second and last volume of Professor Goursat's treatise on the elements of analysis is devoted to the theory of functions of one and of several complex variables, pages 1–305, and to differential equations, total and partial, pages 306–589, concluding with a chapter of forty-two pages on the calculus of variations. Like the first volume, the book is characterized by the vital relation of the choice of material and the mode of treatment to the analysis of the present time. It is the product of a man whose contributions to analysis are of value and who wishes to give to his students the tools he has found to be useful. The procedure that is simplest in practice is given the most prominent place in the presentation, and the main theorems of the subject treated are set in a strong light. On the other hand, a wide range of topics is treated, an outline of a whole theory being sometimes given well on in a chapter as an application of the foregoing principles. When this is done, the author shows good judgment in what he expects of his readers. It is reasonable to leave to the student at this stage the details of proofs which follow general well-defined lines, provided that the framework is so constructed that he can see what is required. The tact which the author displays in this regard makes the book a serviceable one for advanced students to read by themselves and to report on to their university teacher.

The volume begins with an introductory chapter of seventy-four pages, Chapter XIII, on analytic functions of a complex variable, in which both single and multiple-valued functions are treated, the elementary functions being defined in a natural manner for complex values of the argument. The evaluation of integrals of the type

$$\int R(x)dx, \quad \int R(\sin x, \cos x)dx,$$

where R is a rational function, is taken up, and the subjects of infinite series and products, conformal mapping both of plane

and of curved surfaces, with applications to Mercator's chart and stereographic projection and to the transformation of an anchor ring on a rectangle, and families of isothermals, are set forth. In the twenty-five examples that conclude the chapter a number of properties of linear transformations of a complex variable are brought out.

Chapter XIV is devoted to an exposition of Cauchy's theory of functions, with applications to singular points and to Weierstrass's theorems about series of analytic functions. Lagrange's and Burmann's series are treated, the hyperelliptic integrals are studied with reference to questions of periodicity, and a proof that the ratio of the moduli of periodicity of an elliptic integral of the first kind cannot be a real quantity is given, by the aid of Weierstrass's law of the mean for complex integrals, without breaking the integral up into its real and its pure imaginary part. Applications to spherical harmonics appear repeatedly.

Chapter XV, on single-valued functions, contains the theorems of Weierstrass and Mittag-Leffler. The development of $\cot x$ is obtained by the method of residues, the contour of integration being a rectangle. Fifty-eight pages are then devoted to the theory of doubly periodic functions and their inverse, with applications to algebraic curves of deficiency unity.

Chapter XVI, on analytic continuation, and Chapter XVII, on analytic functions of several variables, conclude the part of the volume devoted to the theory of functions. The definition of an analytic function of two independent variables, given on page 264, is as follows: Let A and A' be two regions of the z-plane and the z'-plane respectively and let $f(z, z')$ be uniquely defined for each pair of values z, z' belonging to these regions. If $f(z, z')$ is continuous and each partial derivative $\partial f/\partial z$, $\partial f/\partial z'$, exists at each such point (z, z') — *i. e.*, if each of the ratios

$$\frac{f(z+h, z') - f(z, z')}{h}, \qquad \frac{f(z, z'+k) - f(z, z')}{k}$$

remains finite and approaches a limit as h and k approach 0, — then $f(z, z')$ is said to be an analytic function of the two independent variables z and z'. Since the publication of the book the investigations of Hartogs in the 62d volume of the *Mathematische Annalen* have shown that the condition of continuity is superfluous, being a consequence of the other conditions. Abel's theorem in the theory of the abelian integrals and Darboux's extension of Lagrange's series find a place in the latter chapter.

Passing now to the subject of differential equations, we find in Chapter XVIII an account of the elementary methods for solving total differential equations, or of reducing such an equation in certain cases to equations of lower order. Some properties of Riccati's equation are discussed in § 369, and in § 377 Euler's equation for the elliptic integrals is treated by means of Abel's theorem.

Chapter XIX, on the existence theorems, is exceedingly well done. First comes the proof of the existence of a solution of a system of ordinary differential equations of the first order and of a similar system of partial differential equations, the method being that of power series and *fonctions majorantes*. The proof in the latter case is especially simple and well arranged. Thus in three paragraphs, §§ 383, 384, and 386, all about existence theorems is contained which the reader needs for the cases of chief interest in what follows. But Cauchy's proof by means of a broken line, and the method of successive approximations are given in full. In § 387 the development of the fundamental properties of the solution of a system of ordinary differential equations begins, — properties which in the older treatments like those of Boole and Forsyth (1885) were assumed without proof and which obviously do not hold without restriction, even in simple cases. For example, according to those views, every differential equation of the first order,

$$\frac{dy}{dx} = f(x, y),$$

admits a solution containing an arbitrary constant,

$$y = \phi(x, C),$$

and the particular solution which assumes the value β when $x = \alpha$ can be obtained by solving the equation

$$\beta = \phi(\alpha, C)$$

for C. Furthermore, it was implied or explicitly stated in those earlier days that any function

$$y = \phi(x, C)$$

which is such that the elimination of C between the two equations

$$y = \phi(x, C), \qquad \frac{dy}{dx} = \frac{\partial \phi(x, C)}{\partial x}$$

leads to the given differential equation is *the* solution, *i. e.*, a solution with the above properties. Now, entirely aside from any questions of singular solutions or peculiar functions, this latter theorem is not true. Consider, for example, the differential equation

$$\frac{dy}{dx} = y.$$

A solution of this equation of the above kind is the following:

$$y = \phi(x, C) = e^{x+C}.$$

Moreover, the equation has a solution which vanishes when $x = a$, namely,

$$y = 0.$$

But the constant C cannot be so determined as to yield this solution.

In the early days of modern analysis crinkly curves and uncanny sets of points were most in evidence. But the real contribution of Cauchy and Weierstrass was to furnish the means of obtaining theorems at the threshold of a great subject like differential equations where no theorems of general validity had hitherto existed and where, therefore, no theorem hitherto known could be applied with security in any special case. The French early perceived this phase of progress in analysis, and the book before us is written in this spirit.

I know of no subject, unless it be the study of polynomials and elimination in algebra, that is better suited to give to the student who has just covered the elements of the theory of functions of one or more complex variables, power to apply the theory than a careful working out of the elements of the theory of differential equations, such as is outlined in the chapters before us. I use the word *outlined* advisedly. For while Professor Goursat sets forth in bold relief the main theorems of the theory, he does not in all cases develop the details to the point that the beginner must reach. This is a strong feature of the book for purposes of instruction. For example, the theorems about implicit functions have been carefully developed in their proper place, and they are frequently referred to in these chapters. But it happens time and again that the ultimate formulation is left to the student. And it is for this reason that I included those parts of algebra that deal with

linear dependence and elimination, for it is here, where the attention is not distracted by difficulties in the theory of functions, that the student best learns what the problems of elimination really are.*

But I should give a wrong impression of Professor Goursat's work if I should fail to emphasize the wealth of illustration and the skill with which the leading ideas of a whole theory are at times brought out. Take, for example, this same chapter with the arid title: "Existence theorems." Here we find in three paragraphs at the end, entitled respectively: "One-parameter groups," "Applications to differential equations," and "Infinitesimal transformations," the greater part of the fundamental theorems and points of view of the first two thirds of Lie-Scheffer's book on Differentialgleichungen mit bekannten Infinitesimaltransformationen. Moreover, the theorems are here proven or so far formulated that the student can touch bottom. At least the latter statement becomes true if a slight alteration is made, being in substance what is done at the beginning of Lie-Engel, Transformationsgruppen; for thereby some of the difficulties that present themselves at the outset in connection with the conception of a group are satisfactorily met. The suggestion is this, that we restrict ourselves to the neighborhood of one of the points that is to be transformed and to that of the parametric values corresponding to the identical transformation. Thus for the one-parameter group in the plane we should require that the two functions

$$x' = f(x, y, a) \qquad y' = \phi(x, y, a)$$

satisfy the following conditions:

a) They shall both be analytic at a point (x_0, y_0, a_0);

b) If

$$x' = f(x, y, a), \qquad y' = \phi(x, y, a),$$

and

$$x'' = f(x', y', b), \qquad y'' = \phi(x', y', b),$$

then

$$x'' = f(x, y, c), \qquad y'' = \phi(x, y, c),$$

where (x, y) on the one hand, and a and b on the other, are chosen arbitrarily in the neighborhoods of the point (x_0, y_0) and the point a_0, and where, furthermore,

* On this subject see, in particular, Bôcher, Introduction to Higher Algebra, Chaps. III, IV.

$$c = \psi(a, b)$$

is a function analytic in the point $a = a_0$, $b = b_0$;

c) $\qquad f(x, y, a_0) = x, \qquad \phi(x, y, a_0) = y,$

where (x, y) is an arbitrary point of the neighborhood of (x_0, y_0);

d) At least one of the derivatives $\partial f/\partial a$, $\partial \phi/\partial a$ shall not vanish at the point (x_0, y_0, a_0).

From b) and d) it follows that

$$\psi(a, a_0) = a, \qquad \psi(a_0, b) = b,$$

as also, in particular, that $\psi(a_0, a_0) = a_0$. And from c) we infer that the jacobian of the transformation,

$$\begin{vmatrix} \dfrac{\partial f}{\partial x} & \dfrac{\partial f}{\partial y} \\ \dfrac{\partial \phi}{\partial x} & \dfrac{\partial \phi}{\partial y} \end{vmatrix},$$

does not vanish at the point (x_0, y_0, a_0), its value there being unity.

With these restrictions, all the theorems of the text are readily proven *im Kleinen* for the neighborhood of the point (x_0, y_0, a_0). By means of analytic continuation the theorems can then be extended, and thus the difficulties that present themselves at the beginning are reduced to a minimum.*

Chapter XX is devoted to linear differential equations, the equations of classical type being integrated explicitly and the developments of Fuchs in the neighborhood of a regular singular point being deduced. In Chapter XXI non-linear differential equations are studied. This chapter contains a good treatment of singular solutions.

But the author goes too far at times in omitting to state the restrictions which it is intended to impose. For example, consider § 427, which deals with the author's special investigations

*The procedure in the first chapter of Lie-Engel, Transformationsgruppen, is similar, except that there sufficient conditions to insure the existence of the identical transformation and of inverse transformations are not introduced at the outset, this fact being explicitly stated. On the other hand, Lie-Engel are especially careful to call attention to the necessity of employing a sub-neighborhood, $((x))$, $((a))$ in their notation. The omission of this point by Professor Goursat does not contribute to accuracy or clearness.

on congruences of curves. In order to avoid unnecessary generalities he very properly restricts himself to the case of two polynomials

$$F(x, y, z, a, b) = 0, \qquad \Phi(x, y, z, a, b) = 0.$$

But his arguments do not hold for polynomials, as is seen from the example

$$F(x, y, z, a, b) = (y - a)^2 - bx^2,$$
$$\Phi(x, y, z, a, b) = z - b,$$

both of these polynomials being, moreover, irreducible in the five arguments. It is not obvious what restrictions he means to observe. The above polynomials correspond to the system of differential equations

$$\frac{dy}{dx} = \sqrt{z}, \qquad \frac{dz}{dx} = 0.$$

Chapter XXII contains in sixty-three pages an admirable introduction to the elementary theory of partial differential equations. It is true that the proofs are not always carried down to such a point that the beginner will see how to make contact with his theorems about implicit functions; but he must be able to make such formulations for himself if he expects to do anything with modern analysis, and here is a good chance for him to learn his trade by working through in detail what is well defined in the text. Differential equations and simultaneous systems of the first order in three variables are studied and the theory is illustrated geometrically. Cauchy's problem and the method of characteristics are discussed, and the chapter closes with a paragraph on the Monge-Ampère equations of the second order.

The last chapter of the book, on the calculus of variations, gives an account of those problems and methods, with the development of which the school of Weierstrass has occupied itself, strong and weak variations, sufficient conditions, the E-function, fields of extremals, and Hilbert's invariant integral being the catch-words that suggest the developments.

In these two volumes Professor Goursat has given us a standard work on analysis, one that is modern in all respects, and which will undoubtedly be fully appreciated by students of mathematics and physics.

WILLIAM F. OSGOOD.

SHORTER NOTICES.

Calcul Graphique et Nomographie. Par M. D'OCAGNE. Paris, Octave Doin, 1908. xxvi + 381 pp.

Le Calcul Simplifié. Par M. D'OCAGNE. Paris, 1905. 225 pp.

THE first mentioned volume occurs in the division Sciences Appliquées of the Encyclopédie Scientifique, now publishing under the direction of Dr. Toulouse. It forms a part of the section Science du Calcul. The author's object is to develop as an independent discipline the theory of graphical processes of calculation. The work is therefore mainly to give system and revision to previously published material of his own and that of Culmann, Massau, Mehmke, and others. To this end the volume is divided into Livres I and II.

Livre I, Calcul Graphique, is subdivided into two chapters: Arithmétique et Algèbre graphique and Intégration graphique. In reading these two chapters one would perhaps better forget that any but graphical methods are available. Such methods of necessity often involve trying detail and sometimes sag under their own weight. They lead in the last step to the measurement of a segment by a scale. The author begins therefore with a discussion of "Echelles métriques," and easily converts one to the elegance of the "contre-échelle."

The dominating construction of this first book is that for graphic multiplication: "Si, entre deux parallèles, mM et nN à Oy (* * * lignes de rappel) separées par l'intervalle $mm = a$, nous tirons une droite MN de coefficient angulaire b, la différence NH ou c des ordonnées des points M et N est égal à ab. Pour avoir * * * b il suffit * * * ayant pris sur la partie negative de Ox le segment OQ égal à l'unité * * * de porter sur Oy le segment OQ égal à b, pris avec son signe, et de tirer PQ. La droite MN est dès lors parallèle à PQ." This paragraph is typical of the character and detail of presentation of much of the text.

The value of $a_1b_1 + a_2b_2 + \cdots + a_nb_n$ is then constructed by laying off the coefficients a on OX and b as angular coefficients consecutively and then measuring the segment on the nth *ligne de rappel* or ordinate. Such a construction serves also to solve the linear equation

$$a_0 + a_1 z_1 + \cdots + a_{n-1}z_{n-1} + a_n z_n = 0$$

in ∞^{n-1} ways by merely closing the polygon of angular coefficients on OX at a_n.

Again, a system of n linear equations of n unknowns can be solved directly if they form a "système étagé."

$$a_0 + a_1 z_1 = 0,$$
$$b_0 + b_1 z_1 + b_2 z_2 = 0,$$
$$\cdots\cdots\cdots$$
$$l_0 + l_1 z + l_2 z_2 + \cdots + l_n z_n = 0.$$

By a construction due to Van den Berg any linear system may be reduced to the above form by graphic elimination. An alternative method for the general case is also presented.

A graphic solution of the general equation

$$a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n = 0$$

is based on the method of Lill, in which the coefficients are laid off in succession at right angles and the polygon of multiplication closed by trial and error.

A graphic method of multiplying $F(x)$ by x enables one to plot polynomials in x entirely by rule and compass; and graphic interpolation or determination of the coefficients of the parabola Π_n of the n^{th} degree

$$y = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n,$$

so that the graph passes through a large number of plotted points, is skillfully accomplished.

The second chapter on graphic integration aims at the evaluation of

$$\int F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \cdots\right) dx.$$

The reader must be content, however, if the shot is made at very close range and with a firm rest. The graph C with ordinate y is presupposed and is usually a Π_n, also it does not appear advisable to have F much worse than $x^n y$. The integral curve, whose ordinate measures the desired integral, is usually constructed by inscribing or circumscribing approximate polygons. When a Π_n is to be handled, the construction of the simple successive integrals can be made rigorous. One would scarcely resort, however, to graphic methods in such an emergency.

There is included in this chapter a section on the graphic integration of

$$dy/dx = f(x, y)$$

by approximate graph of the integral curves. The construction is dependent upon plotting the "isoclines"

$$f(x, y) = k.$$

Where possible, use may be made of the loci of inflections or cusps and the envelope.

The author has brought order and unity to a large variety of constructions and made available many scattered results. If the reader is disappointed at the calibre of some of the results, it is a solace to remember that they are *strictly* graphical throughout.

Livre II, Nomographie, takes one into a new atmosphere and presents in slightly over two hundred pages the essentials of this recent branch of mathematics. Those who have read the author's "Traité de Nomographie" with a desire for more compact treatment will find it here in excellent form. There is also a clear separation of the subject into its three essential parts: Représentation nomographique par lignes concourantes, * * * par points alignés, and * * * au moyen de points cotés diversement associées."

To give an idea of nomography itself the author's preface is illuminating: "Il est aisé de faire saisir a priori ce que constitue son essence au moyen d'une comparison familière. Toute le monde connait les graphiques au moyen desquels le Bureau central météorologique indique la repartition des hauteurs barométriques à une date donnée. Sur ces cartes figurent trois systèmes de lignes munis chacun d'une certaine chiffraison: les méridiens dont la cote est la longitude, les parallèles dont la cote est la latitude, les lignes, dites isobares, qui unissent tous les points où la hauteur barométrique atteint une même valeur, inscrite comme cote, à côté de chacune d'elles. Voici donc un nombre, la hauteur barométrique, que depend de deux autres, la longitude et la latitude, qui en est fonction, comme disent les mathematiciens, (* * *) *et dont on a la valeur en lisent simplement la cote d'une certaine ligne, l'isobare, passant par le point de rencontre de deux autres lignes*, le méridien et le parallèle, cotées au moyen des valeurs des nombres donnés."

Having set forth the notion of "echelles fonctionelles," viz.,

scales graduated with lengths proportional to $f(z)$ and inscribed with corresponding z, we are introduced to the important nomogram for

$$f_1(z_1)g_3(z_3) + f_2(z_2)h_3(z_3) + f_3(z_3) = 0, \tag{1}$$

which is solved once for all as follows:

Set $$x = f_1(z_1), \quad y = f_2(z_2).$$

Then (1) becomes

$$g_3 x + h_3 y + f_3 = 0,$$

a system of straight lines z_3. The solution for, say z_2, when z_1 and z_3 are known, requires us to read the z_2 on the line passing through the point of intersection of the lines marked z_1 and z_3. The idea of the function scales on the axes is due to Lalanne and was given the name *anamorphose*.

From the fact that the vanishing of a third order determinant determines the concurrence of three straight lines developes the most general equation in three variables

$$\begin{vmatrix} f_1 & g_1 & h_1 \\ f_2 & g_2 & h_2 \\ f_3 & g_3 & h_3 \end{vmatrix} = 0, \tag{2}$$

whose nomogram has three systems of straight lines. This nomographic representation gives rise to a beautiful application of the projective transformation in the plane to obtain any desirable disposition of the straight lines involved.

It may be well to state here that the general problem, when and only when any given equation is representable by any given type or group of types of nomograms, is still in the main unsolved. Fortunately however nomography finds perhaps its greatest use in engineering where a formula can frequently be identified, by inspection, with some standard type. The question is of mathematical interest however and has been successfully treated in part by E. Duporcq and Goursat.

The most general nomogram of *lignes concourantes* which handles an equation of three variables will be one having three systems of inscribed curves. Furthermore if, instead of graduating the X and Y axes with function scales of one variable, we use the equations

$$x = f_{12}(z_1, z_2), \quad y = f_{34}(z_3, z_4),$$

we are in a fair way to realizing a nomogram for

$$F(f_{12}, f_{34}, z_5) = 0.$$

It remains only to adopt a wing nomogram on each axis to segregate their respective pairs of variables. The axes are then said to have each a *binary scale.*

It is to D'Ocagne himself, in 1884, that we owe "Représentation nomographique par points alignés." He points out clearly the origin in the idea of duality and still continues the constructions with line coordinates. The nomogram of three systems of straight lines gives place to one of three *curved scales*, the key to which is to read the third value where the line joining the given two cuts its scale. The line is of course a moveable straight edge. The advantage gained is appreciable with very little practice. For American engineers the idea of duality and use of parallel coordinates will be unattractive, but the results may be obtained entirely without the use of either as the author points out in Remarque II, page 229.

From this point of view the equation (2) is regarded as determining the alignment of the points defined by

$$x = \frac{f_i}{h_i}, \quad y = \frac{g_i}{h_i} \qquad (i = 1, 2, 3).$$

Such equations are classified by introducing the notion of *nomographic order* on which depends the number of curved and straight function scales. This idea of order is not in complete accord with that used by Soreau * and neither definition appears to determine the order of a given equation absolutely. There are other slight but unfortunate discrepancies in the notations of the two authors, for example, in the meaning of the term "*double alignement.*"

A more complete discussion would require us to reproduce figures and extend this review unduly. The book is thoroughly readable and emphasizes fewer unessential features than the larger treatise. A larger number of examples would improve it; for instance in connection with the nomograms of two straight scales and a web of curves. The volume closes with a note on the general theory.

The second volume appearing in the title is a compact treat-

* R. Soreau, "Contribution à la théorie et aux applications de la Nomographie," *Mém. et Comptes rendus de la Soc. des Ing. civils*, 1901 (tiré à part).

ment with many cuts and pictures of the following six topics: 1) Les instruments arithmétiques; 2) Les machines arithmétiques; 3) Les instruments et machines logarithmiques: 4) Les tables numériques (barèmes); 5) Les tracés graphiques; 6) Les tables graphiques (nomogrammes ou abaques). The volume also includes an interesting introduction and many historical notes.

L. I. HEWES.

Theorie der elliptischen Funktionen. Von H. DURÈGE. In fünfter Auflage neu bearbeitet von LUDWIG MAURER. Leipzig, B. G. Teubner, 1908. 436 + viii pp.

IT is difficult to understand on what ground this work can appropriately be called a new edition of Durège's well-known and admired book, since scarcely a trace of the original seems to have been left.

The first edition appeared in 1861, and was followed by successive editions at intervals of ten years or less, until the fourth in 1887, which was hardly more than a reprint of the third. There can be little doubt as to the inadvisability of further revising this work, now over twenty years old in its latest form, and which represents the state of the theory substantially as it was at the time of Jacobi. Since then this field has been transformed by the new theories of Weierstrass and Riemann, and has been more or less modified or influenced by other lines of mathematical activity such as the theory of groups and the Galois theory of equations. Moreover, the recently developed elliptic modular functions and the still more general class of the automorphic functions afford an extension or generalization which has not only placed the elliptic functions themselves in a new light, but has laid stress on their properties when the periods are regarded as additional independent variables.

It would evidently be quite out of the question to engraft all of these new methods and ideas on to the older theory as expounded by Durège, and Professor Maurer has started *de novo* in his treatment without attempting, as far as we can observe, to incorporate any of the material of the older work except that of course the jacobian functions are given their proper share of attention. The Weierstrassian functions and methods, however, predominate, and the influence of the great work of Klein and Fricke on the elliptic modular functions is observable throughout.

The first chapter treats of the elementary theory of the elliptic integrals, their transformation and reduction to normal forms. The second chapter considers the elliptic Riemann surface and the rational functions on it, while their integrals are discussed in the third chapter. The elliptic functions are studied in the next, and their analytic expressions in the fifth chapter. The sixth is devoted to such applications as the rectification of curves, parametric study of cubic curves, elliptic coordinates, the spherical pendulum, and the rotation of a body about its center of gravity. The seventh chapter is a brief resumé of the elements of group theory and the Galois theory of equations; the eighth treats of the division and transformation of the elliptic functions, and the ninth and last chapter deals with the modular functions. The important formulas are collected together at the end for convenience of reference, and a brief index closes the work.

We believe that Professor Maurer's book will prove to be one of the most useful that has been written on this subject. It embraces in a moderate compass a remarkably well considered and comprehensive view of a large and many-sided field.

J. I. HUTCHINSON.

Introduction to Infinitesimal Analysis. Functions of one Real Variable. By OSWALD VEBLEN and N. J. LENNES. New York, Wiley and Sons, 1907.

JUST fifteen years ago the first treatise in the English language on the theory of functions was published. Until then, this theory, which had been cultivated with great assiduity and success for half a century on the continent, was all but ignored by the English-speaking race, and in this country perhaps not half a dozen universities gave instruction in it. What a change has taken place in the few short years which have meanwhile elapsed! To-day all our better universities regularly give courses in this subject and, strange to relate, the most extended and scholarly treatise on the theory of functions of a complex variable is the work of an American.

During these years, a sister theory has come into prominence; we have now not only a theory of functions of complex variables, but also one for real variables. The work under review treats of this younger theory. Its two hundred and twenty odd pages contain a comprehensive and scholarly presentation of the foundation of the calculus, which on all sides is marked by

originality of treatment and entire mastery of this intricate and difficult subject. The aim of the authors has been to write a book of moderate size which may be used as the basis of a short course or as a reference book in connection with a lecture course on this subject. The book has many praiseworthy features. Besides being very compact, the reasoning is entirely rigorous and correct, a point which, in view of the lax methods in vogue, cannot be too highly commended. The Borel-Lebesgue theorem on point sets included within intervals or rectangles is employed to establish uniform continuity and similar properties. We note as worthy of mention also the following features: the simple manner of introducing irrational numbers with the corresponding geometrical theory of points on the right line; the methodical use of the upper and lower bounds of a function for simplifying proofs, an extended comparison of functions with regard to magnitude (order, rank) with application to indeterminate forms and very general criteria for the convergence of improper integrals, finally the excellent treatment of integration which develops a surprising amount of the theory in a very small compass.

One who has carefully read this little volume will have acquired not only a goodly supply of facts, but, what is of far greater importance, familiarity with the exact and rigorous reasoning which is a sine qua non of further work in higher analysis.

JAMES PIERPONT.

An Algebra for Secondary Schools. By E. R. HEDRICK. American Book Company, 1908. x + 421 pp.

IT is with pleasure that we welcome this little volume into the already large family of elementary algebras. In these days of rapid changes, when so much that is time honored must make way for the new, there is always room for one more algebra, provided it is written like Professor Hedrick's in a thoroughly modern spirit which seeks to readjust both the material chosen and the manner of its presentation so as to satisfy more nearly the needs of the present day.

The book has many features well calculated to win support among progressive teachers. The style is clear and attractive. The genial personality of the author cannot be swallowed up even in a high school algebra. It pervades the whole book and makes it bright and fresh. Without departing too far

from established usage, he has succeeded in giving several subjects a new aspect. As far as possible dead wood has been cut out and live material put in its place. The large number of new and attractive problems drawn from everyday life and from the elements of mensuration and physics will be greatly appreciated by the teacher. Considerable prominence has been given to graphical methods. The author has used them very effectively in treating proportion and variation, in the solution of equations, in establishing the notion of roots and logarithms, and even in explaining interpolation in a logarithmic table. Graphical methods not only make these subjects clearer and more real to the pupil, but will prove of great service to him whether he stops study with the high school or not. In the appendix some of the more advanced topics are treated as permutations and combinations, the binomial theorem, limits and infinite series, irrational and complex numbers.

It needs hardly to be added that the reasoning is sound and in accordance with present-day standards. The pupil will have no vicious habits of thought to eradicate if he goes on with his mathematics. In this connection we may note the emphasis the author lays on checking the solution of equations, a point too often entirely overlooked.

It is a good sign for the future of mathematics in this country when men of the calibre of Professor Hedrick are willing to take time from their own researches to write a work of this character. We wish his little book all success.

JAMES PIERPONT.

Die Lehre von den geometrischen Verwandtschaften. Erster Band: die Verwandtschaften zwischen Gebilden erster Stufe. By RUDOLF STURM. Leipzig and Berlin, B. G. Teubner, 1908. xii + 415 pp.

THE treatise on geometric transformations, of which the present volume is the first part, is estimated to consist of four volumes, the manuscript of which is all in the hands of the printer. The others will be published as soon as possible.

While the first volume is concerned with forms of one dimension, it is not simply a text-book on projective geometry, but considers multiple correspondences, general involutions, trilinearity, and the problem of homography in two and in three dimensions. The first half of the book is concerned with parametric representation, anharmonic ratios, perspectivity, linear

substitution, duality, involution, and polarity. Much stress is laid on directed elements, and all parametric representations are founded on the anharmonic property. The general method is synthetic, but so much use is made of algebraic substitutions that a long article (seventeen pages) is included, explaining the elements of the theory of invariants. All the steps of every proof are given, and a careful interpretation of each step is made. Indeed it would be easy for a patient student to acquire an accurate and extensive knowledge of many of the chapters of projective geometry by reading this book without a teacher. While the use of anharmonic ratios is by no means so great as was predicted for them a few decades ago, yet in parametric representation, theory of invariants, and theory of linear groups they have a lasting value, and most of these important interpretations are found in this book.

The footnotes give a large number of references, but too frequently these mention only the older literature. The technical words used in the text have become in part obsolete, and frequently have definitions different from those used by other writers. Fortunately the book is provided with an extensive glossary which largely atones for the confusion arising from the use of the older words.

The second half begins with an algebraic treatment of multiple correspondence, similar to that in the first volume of the author's treatise on line geometry,* but much more extensive. Halphen's precise application of Carnot's theorem is made the basis of the derivation of Plücker's formulas, and of the proof of the invariance of the genus under birational transformations.

The chapter on the generation of multiple correspondence is rather too abstract. This most useful tool in the study of curves and ruled surfaces frequently offers great difficulty to the reader, and a systematic exposition of the theory should be made as concrete as possible.

The word involution is used to define the usual involution, as well as symmetric (n, n) correspondences which are not involutional. An instructive example of multiple coincidences is followed by the warning to look out for very serious trouble in more complicated cases.

The existence of various involutions and multiple correspondences on the conic is treated in a synthetic manner, with

* Die Gebilde ersten und zweiten Grades der Liniengeometrie in synthetischer Behandlung, Leipzig, Teubner, 1892; vol. 2, 1893; vol. 3, 1896.

the occasional introduction of an equation, which makes the theorems much clearer. In this regard, most of the treatment is less formal but more easily understood than the corresponding part of Reye's treatise.

Since the author holds strictly to the traditions of the Steiner school (though never advocated by Steiner), no figures appear in the book. In many cases the explanations are sufficiently clear without this help, but in discussions of pencils of collineations, products of involutions, and similar subjects, the text could be made much briefer by the addition of appropriate figures.

The fifteen pages devoted to the twisted cubic curve contain a large number of results, skilfully arranged. The chapter on trilinearity is much more systematic than the treatment given this important subject in other places. A number of equations are used to establish the existence of neutral pairs, singular elements, and principal points, but much of the discussion is synthetic. The theory of the cubic surface follows easily from the elementary principles of trilinearity.

Nearly a hundred pages are occupied with the so-called problem of projectivity, *i. e.*, given a series of arbitrary points A_k, and their associates B_k, for any point P to find the corresponding point Q such that the pencil QB_k is projective with PA_k. The problem has no meaning if the number of arbitrary points is less than four, and becomes impossible when it is greater than seven. In particular, the Cremona transformation of degree five having six distinct base points results from five points A_k. The corresponding problem in space includes the discussion of the tetrahedral complex, and some surfaces belonging to it.

Some birational transformations are found in space which are believed to be new.

The later volumes are to contain the theory of collineations in space of two and of three dimensions, including the singular or degraded forms, and the theory of Cremona transformations and multiple correspondences. VIRGIL SNYDER.

Beiträge zur Theorie der linearen Transformationen als Einleitung in die algebraische Invariantentheorie. By W. SCHEIBNER. Leipzig, B. G. Teubner, 1907. 250 pp.

THE contents of most of this book appeared in the *Sächsische Berichte* from 1903 to 1907. The author explicitly declares

that no new results are obtained and no essentially new method developed, but hopes by the repeated and consistent use of one method to awaken further scientific interest. The subject is developed along lines not too difficult for less mature readers, yet the point of view is sufficiently broad to justify the labor and patience involved in working through the maze of long algebraic processes. No use is made of symbolic notation, except that occasionally the author introduces various abbreviations of his own. A more remarkable restriction is that to non-homogeneous forms. This requires frequent paraphrases and the introduction of numerous unsymmetric formulas that are harder to master than the rather simple idea of homogeneous forms.

The ideas of invariant, covariant, source, hessian, jacobian, polarization, and transvectant are clearly treated in the first thirty pages, thus rapidly introducing the reader to the essence of the subject. The application to cubic and quartic forms is similar to that found in the treatise of Burnside and Panton, though more extensive and more systematic. A short chapter discusses the reduction of elliptic differentials to the canonical form; it is more direct than the procedure frequently followed, but presupposes greater maturity on the part of the reader.

Chapter IV, of 42 pages, is devoted to the determination of the system of invariants and covariants of quintic and sextic binary forms. This is the most interesting chapter in the book, although the goal is not always held clearly in view. The various steps are clear and follow the scheme laid down in the preceding chapters quite closely. The proof of the finiteness of the complete system is not completed, except that it is shown that the number of forms for the quintic cannot exceed twenty-three, and these are shown to be distinct by calculating the sources.

At times a paucity of words and a generous amount of algebraic detail lessen the pleasure of reading, but the subject matter makes any other procedure difficult. Unfortunately no geometric interpretation of the various forms is given, and the entire development proceeds strictly along algebraic lines.

The general subject matter of the book is followed by four appendices which are either applications of the methods discussed, or are concerned with linear transformations of non-algebraic functions. The first, of 10 pages, gives the geometric interpretation of the bilinear transformation in the complex

plane. It is a distinct digression from the rest of the book, and is quite elementary. It was introduced to make the later appendices intelligible, but the treatment in these applications is so condensed that it can be of little value to a reader not having much more familiarity with conformal representation than that provided in the first appendix.

The second, of 20 pages, on the Tschirnhausen transformation, includes a detailed treatment of the reduction of the quintic to the Bring-Jerrard normal form; otherwise it is rather similar to the corresponding portion of Weber's Algebra.

The third appendix of 20 pages considers the solution of the icosahedron equation. The first half gives a very rapid survey of the Schwarzian derivative, the hypergeometric series, and the expression of the constants of transformation by means of gamma functions. The subject proper of this appendix is the working out of the problem suggested in Klein's Ikosaeder, page 139, *i. e.*, to start with the general binary quintic and deductively obtain the solution in terms of modular functions. The discussion is followed by a numerical illustration which greatly adds to its clearness.

The last appendix, of 40 pages, is concerned with linear transformations of elliptic theta functions and modular functions. As in the two preceding appendices, the amount of presupposed knowledge is much greater than in the book proper. The treatment is entirely transcendental, and has nothing in common with the preceding portions of the work. Numerous references show the relations between the present development and the existing literature, but it is not clear why this subject should be treated in a book on algebraic invariants.

VIRGIL SNYDER.

Table de Caractéristiques relatives a la Base 2310 des Facteurs premiers d'un Nombre inférieure a 30,030. By ERNEST LEBON. Paris, Delalain Frères, 1906. 32 pp.

IN this pamphlet the author has published a table of "characteristics with respect to the base 2310" by means of which any number between 1 and 30,030 can be readily factored. The first twelve pages are devoted to an explanation of the simple theory upon which the usefulness of the table is based, and to a description of the devices by means of which the characteristics are calculated. The last twenty pages contain the table itself. In a recent number of the BULLETIN, Professor

Fite has translated another article by Lebon in which the computation and use of a larger table, with base $30{,}030 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$ instead of 2310, are described.*

In the factor tables of Glaisher, the numbers are listed and opposite to each is given the smallest prime divisor of the number. Lebon's idea is to divide the numbers prime to a given base B, *e. g.*, $B = 30{,}030$, into classes of the form

$$KB + I \qquad (K = 0, 1, 2, \cdots)$$

according to the values of their residues I with respect to B. The possible prime divisors D less than B are listed, and opposite to each and under each I is placed the characteristic k, if there is any, which specifies the smallest number divisible by D of the class belonging to I. It can then be readily discovered whether or not any other given number in the I-class is divisible by D, since for such a number $K - k$ must be divisible by D.

In the article which is the subject of this review, the characteristics with respect to the base 2310, for prime divisors from 13 to $\sqrt{30{,}030}$ have been listed, so that the table can be applied to the factorization of numbers between 1 and 30,030. If the characteristics for prime divisors up to 2310 had been given, the table would have been applicable to numbers as large as $(2310)^2 = 5{,}336{,}100$. The table described in the article translated by Professor Fite could be used to factor numbers between 1 and $(30{,}030)^2 = 901{,}899{,}900$. The factor tables already published by others, according to Lebon's statement in his introduction, give the factors of numbers from 1 to 10,000,000. All except those by Glaisher for the fourth, fifth, and sixth millions are out of print.

G. A. BLISS.

Serret's Lehrbuch der Differential- und Integralrechnung. Dritte Auflage, neu bearbeitet von GEORG SCHEFFERS. Zwei Bände: I, xvi + 624 pp.; II, xiv + 585 pp. Leipzig, B. G. Teubner, 1907.

SERRET'S name, which is in the title of this well known work, bears about the same relation to the edition under review that Webster's name bears to the latest edition of Webster's dictionary.

* "Theory and construction of tables for the rapid determination of the prime factors of a number," BULLETIN, vol. 13 (1906–7), p. 74. The original article appeared in the *Comptes Rendus*, vol. 151 (1905), p. 78.

The original "Cours de calcul différentiel et intégral," appeared in the seventies and Harnack's translation into German was published in 1884. In 1897–99 a revised edition of this translation by Bohlmann appeared. This second edition, while perhaps the best-known text-book on the calculus in Germany, suffered greatly from the many typographical and mathematical errors. The figures were poorly drawn, many of them being altogether wrong. An incomplete list of over two hundred of these errors was published in connection with the volume on differential equations which was the third volume of the work.

This third edition, by Georg Scheffers, is not only revised, but entirely rewritten and in part rearranged. All the figures have been drawn anew and many new figures added. The theorems have been stated more clearly and more use has been made of italics to make them stand out from the body of the text. Many more references to previous paragraphs have been inserted.

Some of the more important changes which have been made in this edition are the following. The articles on number have been remodeled according to Dedekind's theory and the proofs of the theorems on continuous functions and on the convergence of series have thus been given a real backbone. A new chapter on implicit functions, with a thorough discussion of the functional determinant and of the independence of functions and of equations has been inserted. The difficulties of maxima and minima of functions of several variables receive considerable detailed discussion and the limitations of the parameter method of discussing an extreme value are clearly shown. Many improvements and corrections have been made in the chapters dealing with the application of the calculus to geometry. For instance, the plus or minus sign in the expression for the curvature of a space curve has been removed and in all cases where the square root occurs, a discussion of its sign has been given. The chapter on the complex variable has been entirely rewritten and shortened, the subject matter being limited to just what is necessary for the beginner to read understandingly the remainder of the work and the volume on differential equations which is now in preparation. The rather weak proof of the fundamental theorem of integral calculus in the second edition has been revised and completed. A proof of the theorem, "If $f(x, a)$ is a continuous function of x and a, then the integral

$$F(z, a) = \int_{x_0}^{\infty} f(x, a)dx$$

is a continuous function of z and a," is added to the chapter on definite integrals.

Harnack's appendix on Fourier's series and Fourier's integral has been kept unchanged, but several pages of explanatory notes have been inserted in the form of an introduction to the appendix. The valuable notes and references to other works, given at the end of the volumes in the second edition, have been omitted in the third edition for the rather insufficient reason that they might discourage the student.

This calculus is a geometer's calculus. Over three hundred of the twelve hundred pages are devoted to applications to geometry. With the exception of the paragraphs on center of gravity, there are practically no references to mechanics or physics. All the problems given are worked out in detail. In fact, detail is one of the features of the work. The reviser rejoices in saying that as far as possible he has eliminated from the text such phrases as "the reader will easily see," "the proof is left to the student."

Profiting by the example of the second edition, this edition is quite free from typographical errors. The few found by the reviewer are unimportant. But there is one which may be misleading. The symbol Δ in the second volume is not from the same font of type as that in the first volume, and the difference in the two is sufficient to cause confusion to a beginner.

A detailed table of contents and a copious index make the work very valuable as a reference book, and Serret's Lehrbuch will no doubt continue to be one of the most used books on the shelves in the mathematical reading room at Göttingen.

A. R. CRATHORNE.

Vorlesungen über mathematische Statistik (*Die Lehre von den statistischen Masszahlen*). Von ERNST BLASCHKE, Professor an der Technischen Hochschule in Wien. Leipzig, Teubner, 1906. 263 pp.

THE present work has to do with the theory and application of statistical constants (statistische Masszahlen). It opens with views of different writers as to the field to be included under mathematical statistics. While it contains but little that is new in the line of theory, nearly every point is accompanied by an

apt illustration drawn from actual statistics. This with the many references to the literature makes the book very useful. The probability of life is kept in the foreground throughout. The author discriminates between mathematical and statistical probability, and holds that the two have a priori nothing in common. He gives methods of testing the identity of mathematical and statistical probability, that is to say, whether deviations are such as should be expected in the taking of a random sample.

The author divides statistical constants (Masszahlen) into intensive and extensive. Statistical probability belongs to the former and the various kinds of averages to the latter. On page 74 is the statement that the mode (dichteste Werte) is far the most important of the extensive statistical constants. The justification of this statement would be of interest when we consider it in connection with the almost universal acceptance of the arithmetic mean as a statistical average.

The representation of mortality by the formulas of Moivre, Lambert, Wittstein, and Babbage are classed as empirical representations, while the formula of Makeham is classed as an analytic representation. The chapter on the adjustment of observations contains in a clear form the methods of moments and least squares for fitting curves to observations; and criteria for the critical examination of the best adjustment.

Taken as a whole, the book is a useful contribution to that portion of mathematical statistics which finds its application in the probability of life.

H. L. RIETZ.

Elektrische Wellen-Telegraphie, vier Vorlesungen gehalten von J. A. FLEMING. Autorisierte deutsche Ausgabe von E. ASCHKINASS. Leipzig, B. G. Teubner, 1906. 185 pp.

IN 1903 Fleming delivered the Cantor lectures before the London Society of Arts. These lectures on wireless telegraphy were for a popular audience and necessarily were descriptive and general, not mathematical. The translation by Aschkinass enables German readers to follow the author in their own language — which is very important for the class of readers for whom the lectures are primarily intended. Perhaps the most noteworthy characteristic of the style which is adopted is the constant close touch with nature and the forcefulness of analogy between disturbances in the ether, with which most of Fleming's

audience were probably but poorly acquainted, and the disturbance due to sound in air with which they were certainly much more familiar. This method of exposition is typically English; it is found constantly in the writings of Faraday, Kelvin, J. J. Thomson, and other British physicists; it is met but rarely in the writings of French and German investigators even when they give popular expositions of physical science. Although analogies and mechanical models may be carried to a point where they confuse instead of enlighten and in many cases serve but to disguise the vital truth at the bottom of phenomena, it is probable that a greater use of them would be not without advantage to readers other than English. The translation of Fleming's lectures is therefore undoubtedly of considerable value to German students entirely apart from the matter which the lectures contain.

The first lecture proceeds at once to the presentation of electric force, magnetic induction, electrons, and electromagnetic waves. The analogy between the vibration and pressure in a closed organ pipe and the electric flux and tension on the antennæ of a wireless transmitter is set forth with great force. The function of the condenser is explained and numerous cuts illustrating different fundamental parts of apparatus devised for transmitting electromagnetic waves are given. The second and third lectures deal respectively with the chief points of the transmitting and receiving apparatus. The text by exposition and the cuts by illustration give the reader many of the smaller details as well as the general principles of the apparatus used in wireless telegraphy. In the fourth lecture are discussed numerous matters of a practical bearing on the availability of wireless telegraphy for commercial purposes. To all who realize the difficulties of the purely theoretical investigations in the transmission and reception of electromagnetic waves and who are aware of the many points in which the results of practical experience have been forced to come to the aid of the theory, it will at once appear that Fleming's lectures can be of considerable value not only to general readers but even to those who are primarily interested in and tolerably familiar with the purely theoretical side of the subject.

E. B. Wilson.

CORRECTION.

In the report of the sectional meetings of the Fourth International Congress of Mathematicians, published in the October number of the BULLETIN, an error occurs in the abstract of Professor Dingeldey's paper (on page 20 of the report), the third and fourth lines of which should read "... to generate the circle, the equilateral hyperbola, and the parabola ..."

NOTES.

THE annual meeting of the AMERICAN MATHEMATICAL SOCIETY will be held this year at Baltimore, on Wednesday and Thursday, December 30–31, in affiliation with the American association for the advancement of science. President H. S. WHITE's address, on "Bezout's theory of resultants and its influence on geometry," will be delivered at the opening of the afternoon session on Wednesday. The annual election will close on Thursday morning. Abstracts of papers intended for presentation at this meeting should be in the hands of the Secretary as early as December 12.

THE twenty-third regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will be held at the University of Chicago, on Friday and Saturday, January 1–2, 1909. Titles and abstracts of papers intended for presentation at this meeting should be in the hands of the Secretary of the Section, Professor H. E. SLAUGHT, University of Chicago, on or before December 10.

THE American federation of teachers of science and mathematics will hold its annual meeting at Baltimore on Monday and Tuesday, December 28–29. Further details may be had of Professor H. W. TYLER, chairman of the executive committee, Massachusetts Institute of Technology, or of Professor C. R. MANN, Secretary, University of Chicago.

THE opening (October) number of volume 10 of the *Annals of Mathematics* contains the following papers: "On the small forced vibrations of systems with one degree of freedom," by MAXIME BÔCHER; "Two principles of map-making," by J.

K. WHITTEMORE; "Application of Tchebychef's principle in the projection of maps," by G. W. HILL; "The foundations of trigonometry," by A. C. LUNN; "Sufficient conditions for imaginary roots of algebraic equations," by OTTO DUNKEL.

AT the annual meeting of the London mathematical society, held November 12, the following papers were read: By W. BURNSIDE, "On the theory of groups of finite order" (presidential address); by J. E. LITTLEWOOD, "On the Dirichlet series and asymptotic expansion of integral functions of zero order"; by F. MORLEY, "The norm curves on a given base"; by J. O'SULLIVAN, "Satellite curves on a plane cubic"; by W. BURNSIDE, "On the arithmetic nature of the coefficients in a group of linear substitutions"; by E. W. HOBSON, "On the second mean-value theorem of the integral calculus," and "On the representation of a function by means of a series of Legendre's functions"; by H. BATEMAN, "The conformal representation of space of four dimensions with application to geometric optics"; by D. M. G. SOMMERVILLE, "Periodic properties of partitions"; by A. C. DIXON, "The solution of integral equations"; by A. L. DIXON, "The eliminant of three quantics in two independent variables"; by G. H. HARDY, "A note on the continuity or discontinuity of a function defined by an infinite product"; by F. B. PIDDUCK, "The energy and momentum of an ellipsoid electron"; by F. H. JACKSON, "On q-integration," and "On q-transformations of power series"; by T. STUART, "The complete solution in integers of the eulerian equation $x^4 + y^4 = u^4 + v^4$."

THE second annual meeting of the Italian association for the advancement of science was held at Florence, October 18 to 23, under the presidency of Professor V. VOLTERRA. Twenty sections participated at the meeting. In section I, mathematics, the following papers were presented: By L. AMOROSO, "On the extension of Dirichlet's principle to functions of several complex variables"; by T. BOGGIO, "Solution of some problems connected with the potential of a non-homogeneous sphere"; by U. CRUDELI, "Recent researches on the theory of figures of equilibrium of a liquid mass under uniform rotation"; by A. FAVARO, "Galileo's determination of the weight of the atmosphere"; by G. GIANFRANCESCHI, "Recent progress in electrodynamics of moving bodies"; by M. GREMIGNI, "Archimedes's

postulate in the theory of geometric equivalence"; by G. LORIA, "Geometrography and its transformations"; by P. PIZZETTI, "Mathematics of geodesy"; by F. SEVERI, "Double integrals of the first kind belonging to an algebraic variety"; by E. SOMIGLIANA, "On the mechanical representation of some fields of force"; by A. VENTURI, "On the theory of the balance of Eötrös"; by G. VIVANTI, "On the present state of the theory of integral functions."

The next meeting of the association will be held at Padua, in September, 1909.

A GENERAL invitation has been sent to scientific men throughout the world to attend the meeting of the Australasian association for the advancement of science to be held at Brisbane, Queensland, January 11–17, 1909. Papers to be read before the association should be in the hands of the Secretary before December 26, 1908.

THE firm of Martin Schilling in Halle announces the following new mathematical models: Planigraph, by Professor G. KOENIGS, of the University of Paris; Thread model of the discriminant surface of the quintic equation in the normal form $w^5 + 10xw^3 + 5yu + z = 0$, by Dr. MARY E. SINCLAIR, of Oberlin College, under the direction of Professor O. BOLZA, of the University of Chicago; The singularities of space curves, illustrated by nine models in pasteboard, by Miss HELGA LUND, under the direction of Professor H. G. ZEUTHEN, of the University of Copenhagen. The first model is an apparatus for describing a plane by means of linkages; the second is a developable which divides space into five regions, according to the reality of the roots. Points on the cuspidal edge correspond to three equal roots. The third model includes the tangents and osculating planes to all the branches in each of the eight forms of singularities of space curves.

THE following courses in mathematics are offered during the present winter semester. The courses in analytics, calculus, descriptive geometry and applied mathematics in the technical schools are obligatory.

UNIVERSITY OF PARIS.—By Professor G. DARBOUX: Triply orthogonal systems, two hours.—By Professor E. GOURSAT: Theory of analytic functions, two hours.—By Pro-

fessor P. PAINLEVÉ: Motion and equilibrium, two hours.—By Professor P. APPELL: General mathematics, two hours.—By Professor L. RAFFY: Theory of space curves (analytic), two hours.—By Professor H. POINCARÉ: Theory of the tides, two hours.—By Professor J. BOUSSINESQ: Thermodynamic properties of bodies, two hours.—By Professor E. BOREL: Theory of analytic continuation, two hours.

In the Ecole Normale.—By Professor J. TANNERY: Differential and integral calculus, two hours.—By Professor L. RAFFY: Applications of analysis to geometry, two hours.—By Professors E. BOREL and J. HADAMARD: General mathematics, two hours. Biweekly conferences of one hour each will be held by Professors RAFFY, HADAMARD, and P. PUISEUX and by Dr. E. BLUTEL.

BRAUNSCHWEIG.—By Professor R. DEDEKIND: Elements of the theory of numbers, three hours; Theory of probabilities, two hours.—By Professor R. FRICKE: Analytic geometry and algebra, five hours; Differential and integral calculus, five hours; Theory of functions, three hours.—By Professor W. LUDWIG: Descriptive geometry, five hours; Principles of higher mathematics, five hours.

CHARLOTTENBURG.—By Professors E. LAMPE, G. HETTNER and E. HAENTZSCHEL: Analytic geometry and the calculus, with exercises, six hours.—By Professors JOLLES, E. SALKOWSKI and G. SCHEFFERS: Descriptive geometry, eight hours.—By Professor E. STEINITZ: Algebra, four hours; Theory of potential, four hours; Calculus of variations, four hours (with Dr. C. CRANZ).—By Dr. R. FUCHS: Infinite series, two hours; Theory of functions, four hours.—By Professor O. KRIGAR-MENZEL: Theory of electricity, four hours; Theory of heat, four hours.—By Professor E. HAENTZSCHEL: Mechanics, four hours.

DARMSTADT.—By Professor F. DINGELDEY: Calculus, with exercises, six hours; Theory of Fourier series and integrals, three hours.—By Professor J. HORN: Calculus, with exercises, six hours.—By Professor H. WIENER: Descriptive geometry, with exercises, six hours.—By Professor R. MÜLLER: Descriptive geometry, with exercises, six hours; Analytic geometry, five hours.—By Professor L. HENNEBERG: Mechanics, six

hours (two sections).—By Professor F. GRAEFE: History of mathematics, five hours; Review of elementary mathematics, with exercises, six hours.

MUNICH. — By Professor S. FINSTERWALDER: Analytic mechanics, five hours; Seminar, two hours. — By Professor W. v. DYCK: Differential equations, five hours; Seminar, one hour. — By Professor H. BURKHARDT: Mathematics in natural science, three hours. — By Professor L. BURMESTER: Descriptive geometry, with exercises, six hours (two sections). Sections in analytic geometry and the calculus will be conducted by Professors Finsterwalder, v. Dyck and Burkhardt, six hours. — By Dr. W. KUTTA: Review of algebra, six hours; Trigonometry, five hours; Seminar, two hours.

STUTTGART. — By Professor R. MEHMKE: Descriptive geometry, with exercises, six hours; Vector analysis, three hours; Seminar, two hours. — By Professor C. REUSCHLE: Analytic geometry of space, three hours; Selected chapters of plane analytic geometry and theory of invariants, four hours; Differential and integral calculus, with exercises, six hours; Seminar, two hours. — By Professor E. WÖLFFING: Differential and integral calculus, with exercises, six hours; Higher algebra, four hours. — By Dr. W. BRETSCHNEIDER: Review of elementary mathematics, four hours. — By Dr. E. STÜBLER: Elementary algebra and solution of numerical equations, four hours. — By : Trigonometry, four hours.

PROFESSOR CYPARISSOS STÉPHANOS, of the University of Athens, has been appointed rector of the university for the coming year.

PROFESSOR E. STUDY, of the University of Bonn, and Professor H. BURKHARDT, of the technical school at Munich, have declined the professorship of mathematics at the University of Leipzig.

PROFESSOR R. FRICKE, of the technical school at Brunswick, and Professor H. WIENER, of the technical school at Darmstadt, have been given the honorary title of Hofrat; Professor K. HENSEL, of the University of Marburg, has received the title of Geheimer Regierungsrat.

PROFESSOR E. NEUMANN, of the University of Marburg, has been promoted to a full professorship of mathematics.

DR. C. CARATHÉODORY, of the University of Bonn, has been promoted to an associate professorship of mathematics.

DR. ESCLANGON, of the University of Bordeaux, has been promoted to an associate professorship of mathematics.

PROFESSOR R. C. MACLAURIN, of Columbia University, has accepted the presidency of the Massachusetts Institute of Technology.

PROFESSOR C. A. WALDO, of Purdue University, has accepted the professorship of mathematics at Washington University, St. Louis. Professor A. M. KENYON has been promoted to the head professorship of mathematics at Purdue.

PROFESSOR M. B. PORTER, of the University of Texas, is spending a year abroad on leave of absence.

PROFESSOR R. G. D. RICHARDSON, of Brown University, is spending a year's leave of absence in study at the University of Göttingen.

DR. L. A. HOWLAND has been appointed assistant professor of mathematics at Wesleyan University.

DR. R. C. ARCHIBALD has been appointed professor of mathematics at Brown University.

AT the University of Illinois Dr. A. R. CRATHORNE has been promoted to the position of associate in mathematics.

MR. G. R. CLEMENTS, of Williams College, has been appointed instructor in mathematics at Harvard University.

MR. S. A. SCHWARZ and Mr. W. A. WHYTE have been appointed tutors in mathematics at the College of the City of New York.

MR. A. R. JOHNSON has been appointed instructor in mathematics at Rutgers College.

PROFESSOR A. KORKIN, of the University of St. Petersburg, died April 1, 1908, at the age of 71 years.

RECENT catalogues of second-hand mathematical books: W. Junk, Berlin W. 15, Kurfürstendamm 201, bulletin no. 6, about 400 titles in mathematics. — Oswald Weigels Antiquarium, Leipzig, Königstrasse 1, catalogue no. 132, about 1500 titles in exact sciences.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

ALTERAUGE (L.). Ueber lineare Relationen zwischen hypergeometrischen Integralen. (Diss.) Strassburg, 1907. 4to. 32 pp.

ASKWITH (E. H.). The analytic geometry of the conic sections. New York, Macmillan, 1908. 12mo. 14 + 442 pp. Cloth. $2.60

BOUTROUX (P.). Leçons sur les fonctions définies par les équations différentielles du premier ordre, professées au Collège de France. Avec une note de P. Painlevé. Paris, Gauthier-Villars, 1908. 8vo. 190 pp.

BRESLAU (H.). Dirichlets Satz von der arithmetischen Reihe für den Körper der dritten Einheitswurzeln. (Diss.) Strassburg, 1907. 8vo. 52 pp.

CUTRONI (F.). Alcuni teoremi di geometria negli spazi a n dimensioni. Nota II: Gli angoli. Messina, D'Angelo, 1908. 8vo. 17 pp.

DOBRACHOWSKI (M.). Monographie der Kettenlinie. (Diss.) Bern, 1907. 8vo. 54 pp.

FUBINI (G.). Introduzione alla teoria dei gruppi discontinui e delle funzioni automorfe. Pisa, Spoerri, 1908. 8vo. 13 + 416 pp. L. 15.00

GODFREY (C.) and SIDDONS (A. W.). Modern geometry. Cambridge, University Press, 1908. 8vo. 178 pp. Cloth. 4s. 6d.

HEYWOOD (H. B.). Sur l'équation fonctionnelle de Fredholm et quelques-unes de ses applications. (Thèse.) Paris, Gauthier-Villars, 1908. 4to. 113 pp.

HOCHHEIM (A.). Aufgaben aus der analytischen Geometrie der Ebene. 2es Heft: Die Kegelschnitte. Abteilung 1B: Auflösungen. 3te, vermehrte Auflage, bearbeitet von O. Jahn und F. Hochheim. Leipzig, Teubner, 1908. 8vo. 106 pp. M. 2.20

HOCHHEIM (F.). See HOCHHEIM (A.).

HUPPERT (R. W.). Analytische Untersuchung der allgemeinen Schraubenregelfläche. (Diss.) Strassburg, 1907. 8vo. 31 pp.

JAHN (O.). See HOCHHEIM (A.).

KOWALEWSKI (G.). See NEWTON.

KUMMER (A.). Ueber eine Gattung von projektiven Transformationsgruppen in sechs Veränderlichen. (Diss.) Bonn, 1908. 8vo. 65 pp.

LAUMANN (T.). Ueber den Isomorphismus von Gruppen linearer Substitutionen mit reellen und mit komplexen Koeffizienten. (Diss.) Göttingen, 1907. 8vo. 63 pp.

LINDON (M.). Formeln aus der Differential- und Integralrechnung. Als Anhang zur Sohnckeschen Aufgabensammlung zusammengestellt. Jena, Schmidt, 1908. 8vo. 42 pp. M. 1.50

LOEWENBERG (G.). Was muss man von der analytischen Geometrie wissen? Berlin, Steinitz, 1908. 8vo. 80 pp. M. 1.00

MURRAY (D. A.). Differerential and integral calculus. New York, Longmans, 1908. 8vo. 18 + 491 pp. Cloth. $2.00

NEWTON'S Abhandlung über die Quadratur der Kurven (1704). Aus dem Lateinischen übersetzt und herausgegeben von G. Kowalewski. (Ostwald's Klassiker der exakten Wissenschaften, 164.) Leipzig, Engelmann, 1908. 8vo. 66 pp. M. 1.50

PAINLEVÉ (P.). See BOUTROUX (P.).

RUNGE (C.). Analytische Geometrie der Ebene. Leipzig, Teubner, 1908. 8vo. 4 + 198 pp. Cloth. M. 6.00

RUTHINGER (M.). Die Irreducibilitätsbeweise der Kreisteilungsgleichung. (Diss.) Strassburg, 1907. 8vo. 117 pp.

SCHUMACHER (H. A.). Ueber eine Riemannsche Funktionenklasse mit zerfallender Thetafunktion. (Diss.) Strassburg, 1906. 8vo. 56 pp.

SIDDONS (A. W.). See GODFREY (C.).

THOMAE (J.). Vorlesungen über bestimmte Integrale und die Fourierschen Reihen. Leipzig, Teubner, 1908. 8vo. 6 + 182 pp. Cloth. M. 7.80

WRIGHT (J. E.). Invariants of quadratic differential forms. Cambridge, University Press, 1908. 8vo. 90 pp. 2s. 6d.

II. ELEMENTARY MATHEMATICS.

AMALDI (U.). See ENRIQUES (F.).

AUBERT (P.) et PAPELIER (G.). Exercices d'algèbre, d'analyse et de trigonométrie à l'usage des élèves de mathématiques spéciales. Paris, Vuibert, 1908. 8vo. 362 pp.

BALL (W. W. R.). Récréations mathématiques et problèmes des temps anciens et modernes. 2e édition française, d'après la 4e édition anglaise, et enrichie de nombreuses additions par J. Fitz-Patrick. 2e partie. Paris, Hermann, 1908. 16mo. 2 + 364 pp.

BARDEY (E.). Algebraische Gleichungen nebst den Resultaten und den Methoden zu ihrer Auflösung. 6te Auflage. Bearbeitet von F. Pietzker. Leipzig, Teubner, 1908. 8vo. 12 + 420 pp. Cloth. M. 8.00

BARNARD (S.) and CHILD (J. M.). A new algebra. Vol. I, containing parts 1, 2, 3, with answers. London, Macmillan, 1908. 8vo. 384 pp. Cloth. 2s. 6d.

BERGERVOORT (B. M.). Algebra-Regeln. Sexta bis Untersekunda einschliesslich. Bonn, Georgi, 1908. 8vo. 92 pp. M. 1.00

BERNSTEIN (S. und F.). See YOUNG (G. C. und W. H.).

BERSANO (G. B.). Geometria, per la seconda e la terza classe delle scuole tecniche. Torino, Olivero, 1908. 8vo. 187 pp. L. 2.40

BRIDGETT (R. C.). Blackie's elementary modern algebra with answers. London, Blackie, 1908. 12mo. 232 pp. Cloth. 1s. 6d.

BUSCH (F.). See FÉAUX (B.).

CALDARERA (F.). Primi fondamenti della geometria dello spazio seguito alla 1a parte, col titolo: *Primi fondamenti della geometria del piano.* Palermo, Virzi, 1908. 8vo. 4 + 235 pp. L. 7.00

CHILD (J. M.). See BARNARD (S.).

COLLINS (J. V.). Practical elementary algebra. New York, American Book Co., 1908. 12mo. 3 + 420 pp. Half leather. $1.00

COMBEROUSSE (C. DE). Cours de mathématiques, à l'usage des candidats à l'Ecole polytechnique, à l'Ecole normale supérieure, à l'Ecole centrale des arts et manufactures. 5e édition, revue et augmentée. Vol. I, seconde partie: Algèbre élémentaire. Paris, Gauthier-Villars, 1907. 8vo. Pp. 295–796. Fr. 6.00

CORTÁZAR (J.). Tratado de algebra elemental. 34a edición, corregida y arreglada. Madrid, 1908. 48 pp. P. 3.50

DAVISON (C.) and RICHARDS (C. H.). Plane geometry for secondary schools. In four books. Cambridge, University Press, 1908. 8vo. Pp. 136, 85, 143, 81. Each, 1s.

ENRIQUES (F.) e AMALDI (U.). Elementi di geometria, ad uso delle scuole secondarie superiori. 3a edizione. Bologna, Zanichelli, 1908. 16mo. 7 + 599 pp. L. 4.50

FÉAUX (B.). Buchstabenrechnung und Algebra, verbunden mit Aufgabensammlung. 11te, verbesserte und vermehrte Auflage, besorgt durch F. Busch. Paderborn, Schöningh, 1908. 8vo. 6 + 360 pp. M. 2.80

FERVAL (H.). Eléments de trigonométrie. 4e édition, revue. Paris, Hachette, 1908. 16mo. 4 + 296 pp. Fr. 2.50

FITZ-PATRICK (J.). See RALL (W. W. R.).

GRAVE (W.). Beiträge zur rationellen Didaktik mathematischer Disziplinen. (Diss.) Leipzig, 1908. 8vo. 88 pp.

HEDRICK (E. R.). An algebra for secondary schools. New York, American Book Co., 1908. 12mo. 10 + 421 pp. Half leather. $1.00

HUNTINGTON (E. V.). Four-place tables of logarithms and trigonometric functions. Unabridged edition. Cambridge, Mass., Harvard Co-operative Society, 1908. 8vo. Cloth. $0.60

INTERMEDIATE applied mathematics papers; being the questions set at the University of London from 1882 to 1908. London, Clive, 1908. 8vo. 76 pp. 2s. 6d.

LENNES (N. J.). See SLAUGHT (H. E.).

LYDON (N. S.). A preliminary geometry. London, Methuen, 1908. 8vo. 112 pp. 1s.

LYMAN (E. A.). Solid geometry. New York, American Book Co., 1908. 12mo. 16 + 336 pp. Half leather. $0.75

MASSARI (V.). Elementi di algebra practica, ad uso delle scuole tecniche e normali. Roma, Voghera, 1908. 16mo. 4 + 128 pp. L. 1.50

MATRICULATION model answers: mathematics. Being the London University matriculation papers in mathematics from Sept., 1904 to June, 1908. London, Clive, 1908. 8vo. 178 pp. Cloth. 2s.

METZ (C.). Fünfstellige Logarithmen der Zahlen 1–10800 und der trigonometrischen Funktionen. Für den praktischen Gebrauch eingerichtet. Ausgabe A mit vollständigem Rand-Index. Berlin, Seydel, 1908. 8vo. 8 + 85 pp. Cloth. M. 4.00

MILNE (W. J.). Standard algebra. New York, American Book Co., 1908. 12mo. 464 pp. Cloth. $1.00

NEVEU (H.). Cours d'algèbre á l'usage des classes de mathématiques élémentaires, des candidats á l'Ecole de Saint-Cyr et au professorat des Ecoles normales. 3e édition, revue et corrigée conformément aux derniers programmes officiels. Paris, Masson, 1908. 8vo. 671 pp.

NOCIONES elementales de geometría aplicadas al dibujo lineal, seguidas de las soluciones y respuestas, por los Hermanos de las Escuelas cristianas. Paris, 1908. 16mo. 31 pp.

PAPELIER (G.). Lee AUBERT (P.).

PIETZKER (F.). See BARDEY (E.).

RICHARDS (C. H.). See DAVISON (C.).

SACHS (J.). Tafeln zum mathematischen Unterricht. Leipzig, Teubner, 1908. 8vo. 2 + 120 pp. M. 6.00

SCOONES (P.) and TODD (L.). The Eton algebra, Part I, with answers. London, Macmillan, 1908. 8vo. 210 pp. Cloth. 2s. 6d.

SCOTTI (G.). Elementi di geometria, ad uso del ginnasio superiore. 4a edizione. Torino, Salesiana, 1908. 8vo. 127 pp.

SLAUGHT (H. E.) and LENNES (N. J.). High school algebra, complete course. Boston, Allyn & Bacon, 1908. 12mo. 15 + 494 pp. Cloth. $1.20

SMOLEY (C.). Five-place logarithmic-trigonometric tables. London, Constable, 1908. 8vo. 6s.

——. Parallel tables of logarithms and squares, etc. 5th edition, revised. London, Constable, 1908. 8vo. 14s.

SPIEKER (T.). Lehrbuch der ebenen Geometrie mit Uebungs-Aufgaben für höhere Lehranstalten. Ausgabe C. Abgekürzte Kurse. 4te Auflage. Potsdam, Stein, 1908. 8vo. 4 + 206 pp. M. 2.50

STÜCK (E.). Vierstellige logarithmische und trigonometrische Tafeln. Herausgegeben auf Veranlassung des nautischen Departements des Reichs-Marineamts. Wilhelmshaven, Lohse, 1908. 8vo. 4 + 94 pp. Cloth. M. 1.25

TODD (L.). See SCOONES (P.).

TOURS (F. I. C.). Tables de logarithmes à cinq décimales pour les nombres de 1 à 10,000, et pour les fonctions trigonométriques de minute en minute. Paris, Poussielgue, 1908. 16mo. 8 + 144 pp.

TOURS (F. J.). Eléments de trigonométrie rectiligne avec de nombreux exercices. Paris, Poussielgue, 1908. 16mo. 7 + 257 pp.

WOLFF (H.). Sätze und Aufgaben der Geometrie für Realanstalten, nach den preussischen Lehrplänen von 1901 zusammengestellt. 1er Teil: Pensum der Unterstufe (Quarta bis Untersekunda). Leipzig, Teubner, 1908. 8vo. 4 + 98 pp. M. 1.60

YOUNG (G. C. and W. H.). Der kleine Geometer, Deutsche Ausgabe, besorgt von S. und F. Bernstein. Leipzig, Teubner, 1908. 8vo. 16 + 239 pp. Cloth. M. 3.00

ZORETTI (L.). Leçons d'algèbre, classes de mathématiques A et B, préparation aux Ecoles navales et de Saint-Cyr. Paris, Paulin, 1908. 16mo. 3 + 461 pp. Fr. 6.00

III. APPLIED MATHEMATICS.

BALL (R. S.). Meccanica. Traduzione di J. Benetti. 4a edizione, riveduta. Milano, Hoepli, 1908. 16mo. 16 + 198 pp.

BEDFORD (E. J.). Introductory mechancis; with diagrams. New York, Longmans, 1908. 16mo. 10 + 141 pp. Cloth. $0.50

BENETTI (J.). See BALL (R. S.).

BRILLOUIN (M.). Mémoire sur l'ellipticité du géoïde dans le tunnel du Simplon. Paris, 1908. 8vo. 230 pp.

CALCULATIONS of elements of machine design. New York, Industrial Press, 1908. 8vo. 48 pp. $0.25

CHURCH (I. P.). Mechanics of engineering, for use in technical schools. Revised edition, partly rewritten. New York, Wiley, 1908. 8vo. 26 + 854 pp. Cloth. $6.00

CROSS (C. R.). Notes on mechanics, printed for the use of students in the Massachusetts Institute of Technology. Boston, Cross, 1908. 8vo. 75 pp. (Not for sale.)

CZUBER (E.). Wahrscheinlichkeitsrechnung und ihre Anwendung auf Fehlerausgleichung, Statistik und Lebensversicherung. Vol. I. Wahrscheinlichkeitstheorie, Fehlerausgleichung, Kollektivmasslehre. 2te, sorgfältig durchgesehene und erweiterte Auflage. (Teubners Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen, Vol. IX, 1.) Leipzig, Teubner, 1908. 8vo. 10 + 410 pp. Cloth. M. 12.00.

DIAMANT (S.). Curves for calculating beams, channels and reactions. London, Spon, 1908. 8vo. Cloth. 8s. 6d.

DOBBS (W. J.). Examples in elementary mechanics: practical, graphical and theoretical. London, Methuen, 1908. 8vo. 356 pp. Cloth. 5s.

DONATH (A.). Lehrbuch der Elektromechanik. Eine Darstellung der wissenschaftlichen Grundlagen der Elektrotechnik und ihrer praktischen Anwendung in ausführlicher, leichtverständlicher Form. Zum Gebrauche für technische Lehranstalten, sowie ganz besonders zum Selbstunterricht. Vol. I. Der Magnetismus, der Elektromagnetismus und die Berechnung der Feldmagnete von Dynamomaschinen und Elektromotoren. Mit zahlreichen Rechnungsbeispielen. 2ter Teil. Jena, Costenoble, 1908. 8vo. Pp. 15–18 and 385–731. M. 6.00

FAURE (F.). Eléments de statistique (résumé du cours fait à la Faculté de droit de Paris, 1904–05). Paris, Larose, 1906. 18mo. 8 + 135 pp. Fr. 2.50

FERRY (E. S.). Brief course in elementary dynamics for students of engineering. New York, Macmillan, 1908. 8vo. 11 + 182 pp. Cloth. $1.25

FOLLOWS (G. H.). The essentials of descriptive geometry. New York, Hill, 1908. 8vo. 100 pp. Cloth. $2.00

HENDERSON (P.). Zur Thermodynamik der Flüssigkeitsketten. (Diss.) Göttingen, 1907. 8vo. 50 pp.

JERRETT (H. D.). Hydraulic tables and memoranda. San Francisco, Hicks-Judd Co., 1908. 16mo. 8 + 142 pp. Cloth. $1.50

MÜLLER-BRESLAU (H. F. B.). Die graphische Statik der Baukonstruktionen. Vol. II. 2te Abteilung. 2te Lieferung. Leipzig, Kröner, 1908. 8vo. Pp. 8 + 97–594. M. 15.00

NORWELL (A.). Elementary applied mechanics. • Statics, introducing the unitary system. London, Longmans, 1908. 8vo. 256 pp. Cloth. 3s.

REVUE de mécanique. Tables décennales des matières et des noms, 1897–1906. Paris, Dunod, 1908. 4to. 149 pp.

SANBORN (F. B.). Mechanics problems for engineering students. 3d edition. New York, Engineering News Pub. Co., 1908. 12mo. Cloth. $0.50

SPANGLER (H. W.). Notes on thermodynamics: the derivation of the fundamental principles of thermodynamics and their application to numerical problems. 4th edition, revised. New York, Wiley, 1908. 12mo. 7 + 75 pp. Cloth. $1.00

STUPAR (A.). Lehrbuch der astronomischen Navigation. Im Auftrage des k. k. Reichskriegsministeriums, Marinesektion, verfasst. Wien, Gerold, 1908. 8vo. 16 + 275 pp. Cloth. M. 6.00